CONTROLS

NOTES

DATE : May. 20, 2007.

EE

7.00 - 1.00 ⇒ Control systems → Hall 7

2.30 - 8.30 ⇒ Digital electronics → Hall 7.

21-05-07

CONTROL SYSTEMS → 15 Marks.

1. Nagrath & Gopal.
2. B.C. KUO
3. IES / IAS papers G.K. publishers.
4. A.K. Jairath

→ T/f, Block diagram, signal flow — 2 M

→ Time Domain Analysis → 4 M {f/b changes the location of poles}

→ stability [RH / RL / BP / NP] → 4 to 6 M } → for closed loop

Compensators (PID controller) → 2 M

→ ~~steady state~~ state space Analysis (Ogata) → 2 to 4 M → Multi i/p, Multi o/p.

Transfer functions

→ order of the system → no. of (or) one time constant storage elements

T/f is a mathematical equivalent Model for a system.

* T/f valid for → Linear time invarient (LTI)

TDA (Time Domain Analysis) → to know about the performance of the system w.r.t. time. {Time domain specifications}

→ for unbounded signals we donot find the stability (ramp)

State space Analysis for → Dynamic systems [linear / Non-linear / time varient / invarient]

→ -ve f/b → poles shifted to left

+ve f/b → poles shifted to right

→ In closed loop systems if order of the system is very high, it is difficult to find roots of T/f. So we use RH, RL/BP/NP

* RH → char. eq to find CL. stability

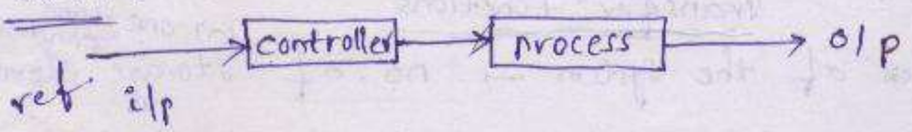
* RL / BP / NP → O/L

* Order → NP, RL, BP, RH.

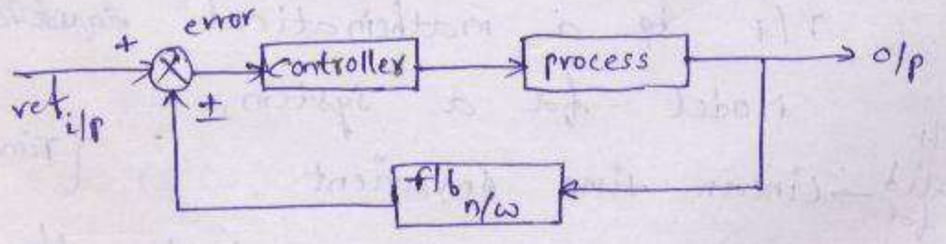
⇒ <u>Control system:</u> It is an arrangement of group of phy. components in such a way that it gives the desired o/p by means of controller. either direct method or indirect.

→ Based on the controller action, control systems
- O/L system
- C/L system.

OLCS :-



CLCS :-



OLCS :- A system in which the controller action is inde. of o/p. Eg :- fans, heater. normal, iron box, traffic lights

Eg :- Any system which not sense the o/p.

CL CS :- The controller action is totally depends on o/p. Eg :- Any m/c with Automatic which sense the o/p. [Refrigerator, Iron box automatic

⇒ **F/b n/w:-** It is nothing but a transducer which converts energy from one form to the another form.

* It consists passive elements R, L, C. The max. value of F/b n/w ratio is one.

→ F/b is the property of the CL system which brings the o/p to the ~~actual~~ i/p ~~is~~ ~~used~~ to compare with ref i/p and generates error signal, then the controller is adjusted such that error becomes zero.

⇒ **T/F :-** It is a mathematical equivalent model for the system.

DEF: A T/F of a Linear time invariant (LTI) is defined as ratio of L.T o/p to L.T i/p. with all initial condi.s are zero.

(low pass → Integrator)

Linear Systems → Transfer function

Non- linear → Decribing function

DEF 2: A T/F of a LTI, is also defined as L.T. of impulse response with all initial condi.s are zero.

* impulse i/p → System → impulse response

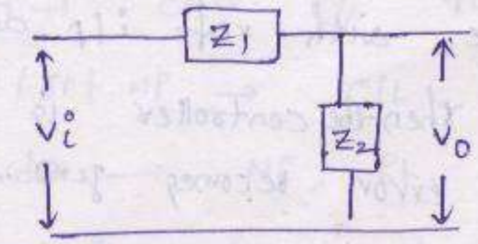
↓

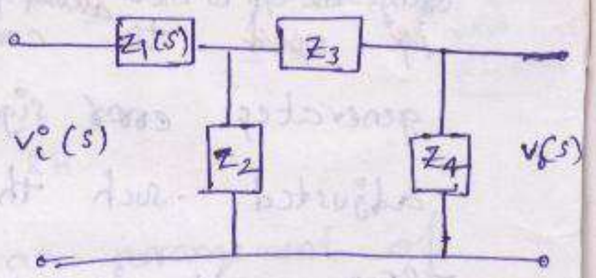
[Natural response or actual System response or free forced response.]

* for, ramp, step → forced response.

⇒ T/f → Electrical n/ω
→ Differential eq.
→ Signal response

→ Electrical n/ω :-



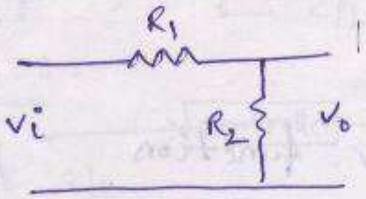


$$\frac{V_0(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{Z_2(S).Z_4(S)}{Z_1(S)\{Z_2(S)+Z_3+Z_4\} + Z_2[Z_3+Z_4]}$$

Q. find the T/f for the following :- and represent poles and zeros in s-plane.

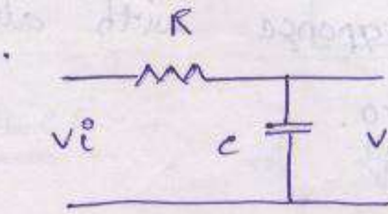
(i).



$$\frac{V_0}{V_i} = \frac{R_2}{R_1+R_2}$$

↑ * attenuation factor
[No poles & zeros]
because no storage elements

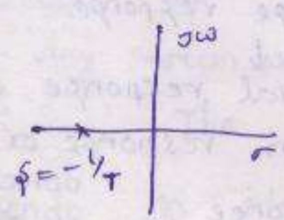
(ii).



$$\frac{V_0}{V_i} = \frac{1/cs}{R + 1/cs}$$

$$= \frac{1}{sCR+1}$$

take $\tau = RC$



$$= \frac{1}{s\tau+1}$$

(first order) standard form

* A pole is nothing but −ve of inverse of system time constant at which the magnitude of T/F is ~~zero~~ infinity.

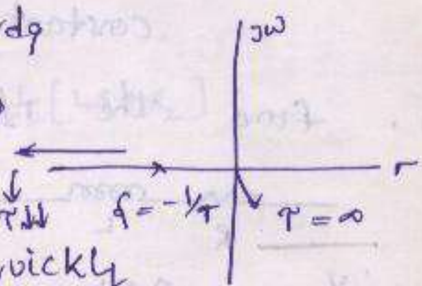
* → Behaviour of the system is given by τ.

* If τ ↑, (large) system response is slow.

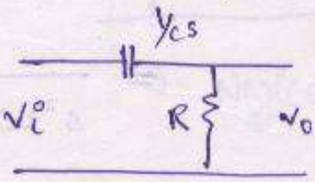
* τ at origin is infinity.

→ τ is nothing but −ve of inverse of dominent pole location $\tau = -1/\text{pole}$.

* As the pole moves towards to the left, the τ is decreased and system reaches steady state quickly and becomes more stable.



(iii).



$$T/F:\ \frac{v_0}{v_i} = \frac{R}{R + 1/Cs}$$

$$= \frac{CSR}{SCR+1}$$

Let $\tau = RC$

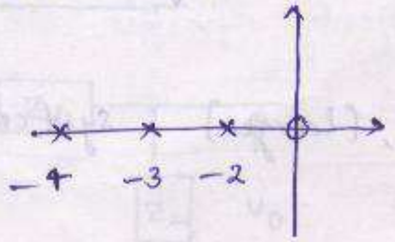
$$= \frac{S\tau}{S\tau + 1}$$

$$= \frac{S}{S + 1/\tau}$$

* By changing the position of components the no. of poles are same and position also same but the no. of zeros changes and ~~zeros~~ position changes.

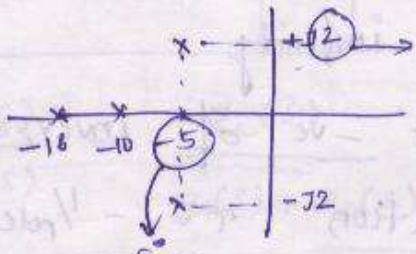
⟹ A zero is −ve of inverse of system time constant at which magnitude of T/f is zero.

(iii). findout time constant,



$$\tau = -\frac{1}{(-2)} = 0.5$$

(iv).

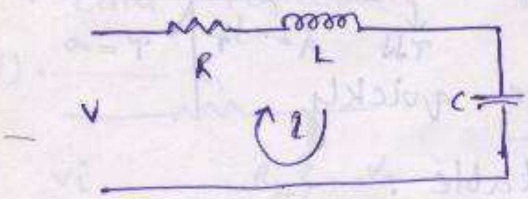


→ freq of oscillations

Time constant

(v). find the T/f.

2 storage elements → 2 order.



$$V(s) = I(s)\left[R + sL + \frac{1}{sC}\right]$$

$$T/f = \frac{I}{V} = \frac{1}{R + sL + \frac{1}{sC}} = \frac{sC}{s^2LC + sCR + 1}$$

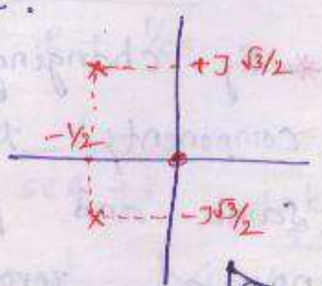
Let $L = 1\,H$

$C = 1\,F$

$R = 1\,\Omega$

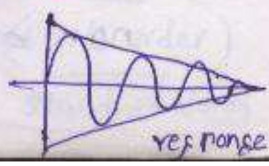
Then locate poles & zeros. and explain what type of response.

$$\frac{I}{V} = \frac{s}{s^2 + s + 1}$$

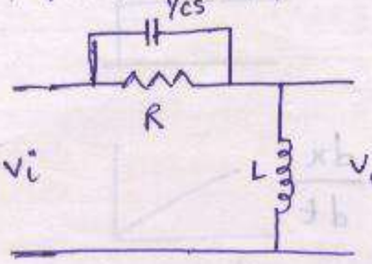


Time constant = 2

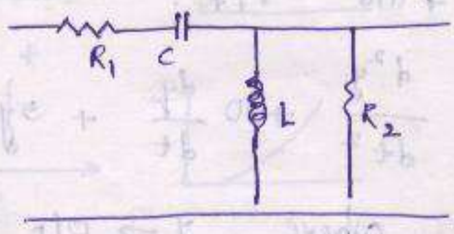
freq of oscillation = $\frac{\sqrt{3}}{2}$ rad



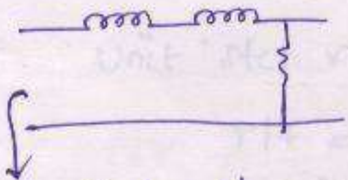
(vi). find T/F.



(vii).

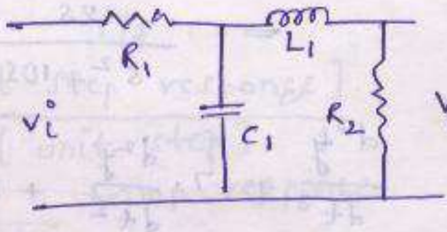


→ for electrical n/w, Modern control system by A.K. Jairath.



Only first order

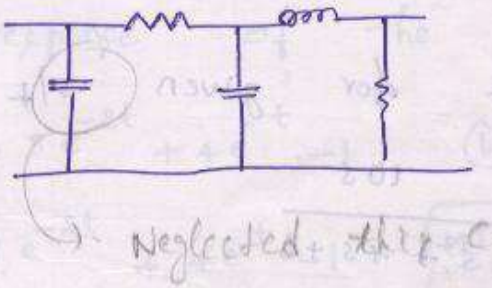
(viii).



Ans. (viii)

$$\frac{V_o}{V_i} = \frac{\frac{1}{cs} \cdot R_2}{R_1\left[\frac{1}{cs} + Ls + R_2\right] + \frac{1}{cs}\left[Ls + R_2\right]}$$

Eg :-



Neglected the capacitance.

→ Differential Equations :- [D.E.]

1. find T/f.

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 5y = 2\frac{dx}{dt}$$

where $y \to$ o/p & $x \to$ i/p

$$\frac{Y(s)}{X(s)} = \frac{\text{i/p related terms}}{\text{o/p related terms}}$$

$$= \frac{2s}{s^2+10s+5}$$

2. $$\frac{d^3y}{dt^3} + 7.\frac{d^2y}{dt^2} + 10 = 5.\frac{d^2x}{dt^2}$$

$$T/F = \frac{5}{s+7}$$

* Here 10 is a initial condi. So in T/f evaluation initial condi-s are zero.

3. Obtain D.E for given T/f.

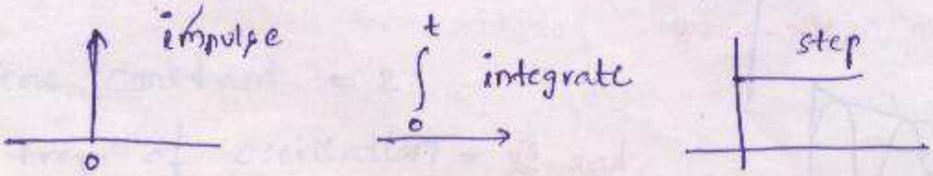
$$\frac{Y(s)}{X(s)} = \frac{10s}{s^2+7s+6}$$

$$\frac{d^2y}{dt^2} + 7.\frac{dy}{dt} + \underset{\downarrow \\ +k}{6y} = 10.\frac{dx}{dt}$$

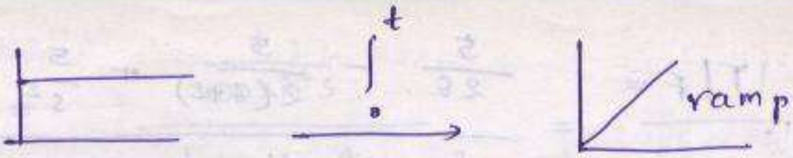
→ Signals Response :-

TF = L[impulse response]

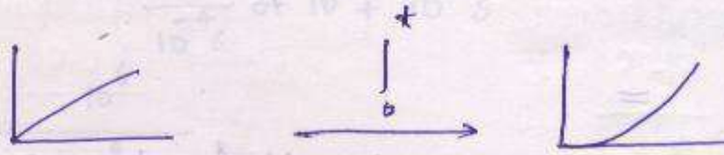
(1).



(2).



(3).



⟹ Types of questions :-

Given	find
(1). Step response	T/F

$T/F = L[\text{Impulse response}] = 0$

(2). Unit step response — T/F

$$T/F = \frac{L[\text{unit step response}]}{L[\text{unit step}]}$$

(3). Impulse response — Ramp response

$$\int_0^t \int_0^t - \, dt$$

1. The unit impulse response of a system is $c(t) = -4 \cdot e^{-t} + 6 \cdot e^{-2t}$, $(t \geq 0)$. The step response of the system is ?

ⓐ. $-3e^{-2t} + 4e^{-t} - 1$ ⓑ. $-3e^{-2t} - 4e^{-t} - 1$

ⓒ. $3e^{-2t} + 4e^{-t} - 1$

(4). Ramp — Step

$$T/F = \frac{L[U.R.R]}{L[U.R.]}$$

Just do integrate.

$$\int_0^t c(t) =$$

2. The unit step response is $\phi(t) = \frac{5}{2} - \frac{5}{2}e^{-2t} + 5t$

The T/F is —?

$$T/F = \frac{L[\text{unit step response}]}{L[\text{unit step}]}$$

$$T/f = \frac{\frac{5}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}}{1/s}$$

$$=$$

3. A system described by,

$\frac{d^2y}{dt^2} + 3 \cdot \frac{dy}{dt} + 2y = x(t)$ is initially at rest, for the i/p $x(t) = 2\,u(t)$. the o/p $y(t)$ is — ?

for response / o/p :-

ⓐ. first find T/f.

ⓑ. substitute i/p

ⓒ. partial fractions.

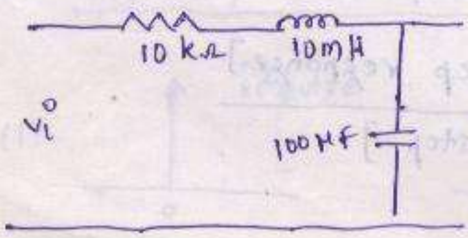
ⓓ. Apply L.T.

$$T/f = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 3s + 2}, \quad X(s) = \frac{2}{s}$$

$$\rightarrow Y(s) = \frac{2}{s(s^2+3s+2)}$$

Ans. $2(1 - 2e^{-t} + e^{-2t})\,u(t)$

4. for the ckt shown in fig. initial condi's are zero. its T/f is — ?



①. $\frac{1}{s^2 + 10^6 s + 10^6}$

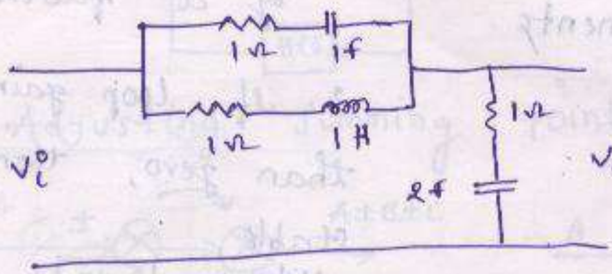
②. $\frac{10^6}{s^2 + 10^3 s + 10^6}$

③. $\frac{10^3}{s^2 + 10^3 s + 10^6}$

$$\frac{V_o}{V_i} = \frac{\frac{1}{100\times10^{-6}s}}{\frac{1}{10^{-4}s} + 10^{4} + 10^{-2}s} =$$

Ans. $\dfrac{10^6}{s^2+10^6 s+10^6}$

5.

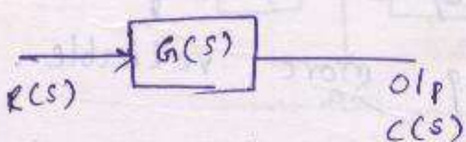


$$\frac{(1+\frac{1}{s})(1+s)}{2+s+1/s} = 1 \quad , \quad \frac{V_o}{V_i} = \frac{1+\frac{1}{2s}}{1+1+\frac{1}{2s}} = \frac{2s+1}{4s+1}$$

⇒ **Block diagram :-**

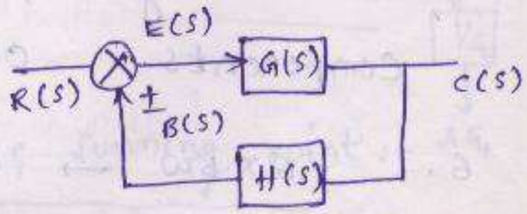
It is a short hand pictorial representation of system b/w i/p & o/p.

OLCS



$$T/F = \frac{C(s)}{R(s)} = G(s)$$

CLCS



$G(s)$ – forward path gain $= \dfrac{C(s)}{E(s)}$.

$H(s)$ – f/b path gain $= \dfrac{B(s)}{C(s)}$

$G(s).H(s)$ – open loop ~~gain~~ T/F

This represents actual system $= \dfrac{B(s)}{E(s)}$

$$CL\ T/F = \frac{G(s)}{1 \mp G(s).H(s)}$$

* Oscillators → +ve f/b → unstable

* Multivibrators → +ve f/b → stable

Comparision :-

OLCS	CLCS
1. NO f/b	1. gain will be reduced by a factor (1+ GH).
2. Less components	2. If loop gain more than zero, then more stable, stability so depends on loop gain
3.	3. Accuracy is depends on the f/b n/w.
	4. Less sensitive with f/b the sensitivity improved, the sensitivity factor is less.

The better is less sensitive.

5. Reliability is depends on no. of components. OLCS is more reliable.

6. G×BW → constant.

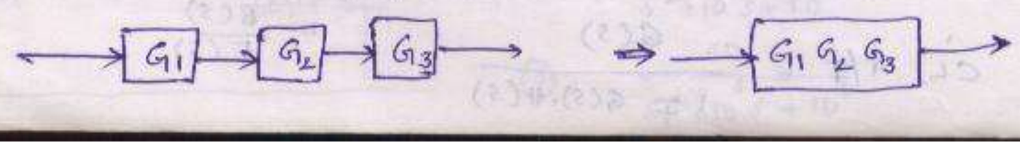
operating area is a bandwidth.

so for CLCS bandwidth is more

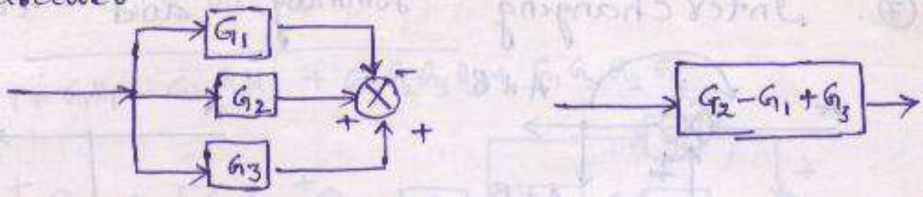
⟹ BLOCK DIAGRAM REDUCTION RULES :-

① Cascade / series

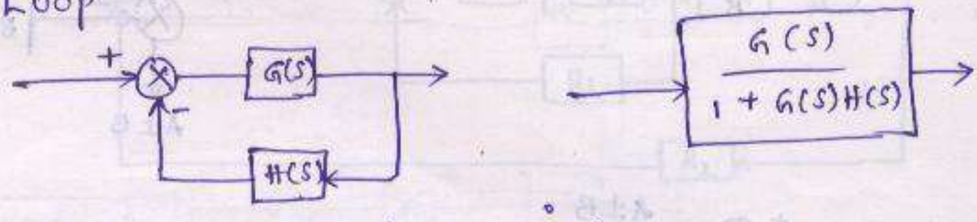
valid for signal flow graph also



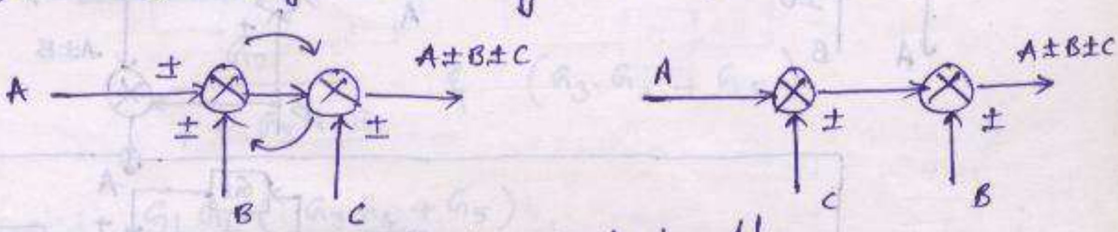
2. Parallel



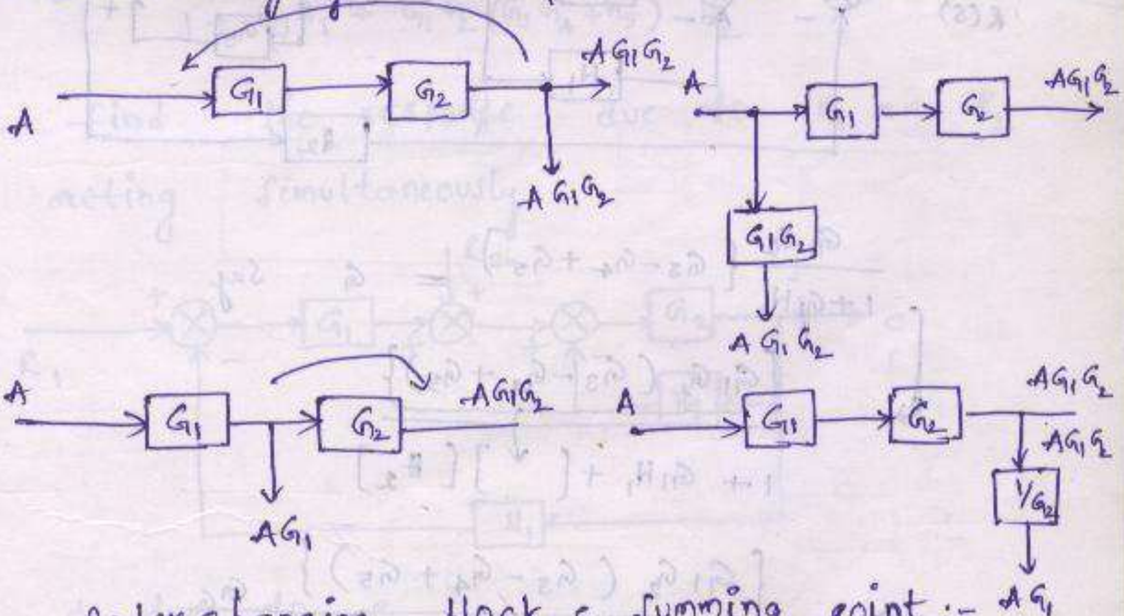
3. Loop



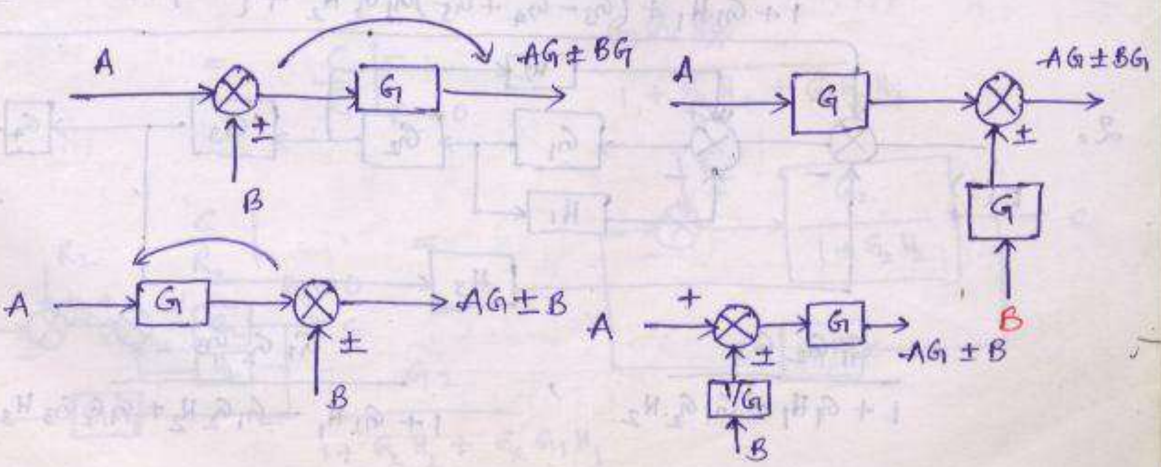
4. Adjusting summing points



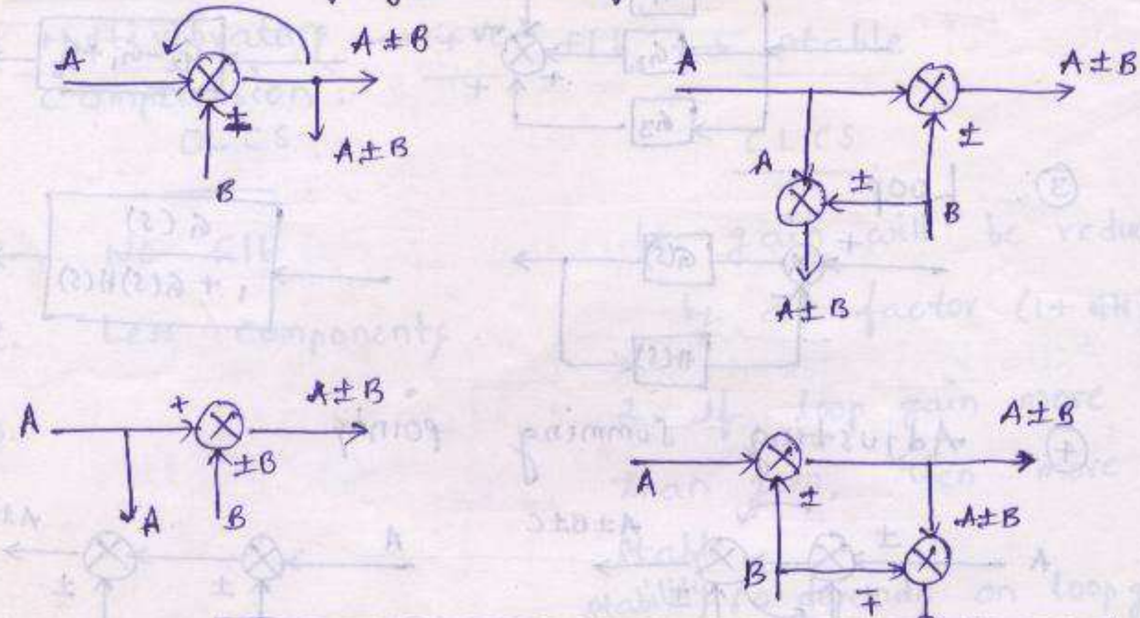
5. Interchanging block & take off :-



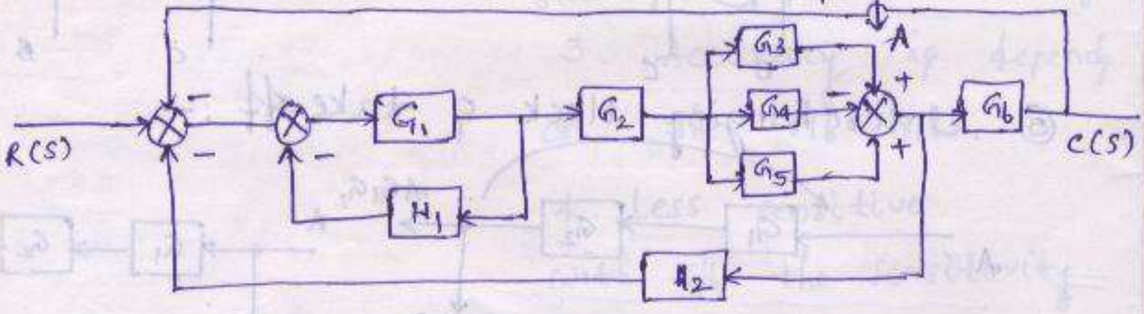
6. Interchanging block & summing point :-



7. Interchanging summing and take-off :-



1.

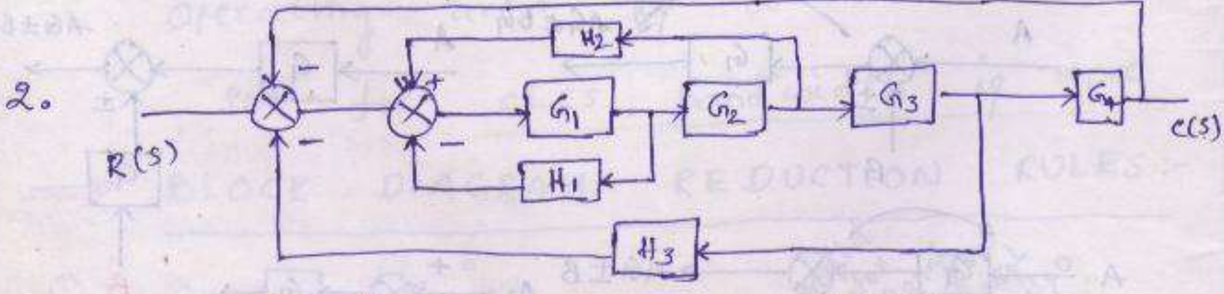


$$\frac{G_1 G_2}{1+G_1H_1}\{G_3-G_4+G_5\} = G \quad \text{say}$$

$$\frac{\{G_1 G_2 (G_3-G_4+G_5)\}}{1+G_1H_1+[\quad][H_2]}$$

$$\Rightarrow \frac{\{G_1 G_2 (G_3-G_4+G_5)\}\, G_6}{1+G_1H_1+(G_3-G_4+G_5)G_1G_2H_2+[\quad]1}$$

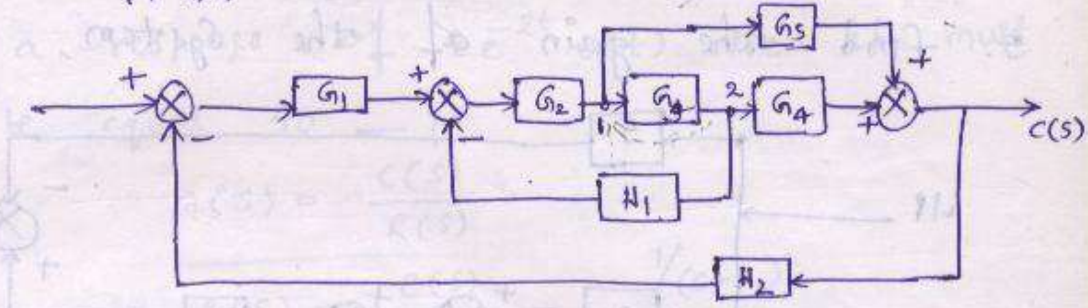
2.



$$\frac{G_1G_2G_3}{1+G_1H_1-G_1G_2H_2}, \qquad \frac{G_1G_2G_3}{1+G_1H_1-G_1G_2H_2+G_1G_2G_3H_3}$$

$$= \frac{G_1 G_2 G_3 G_4}{1 + G_1H_1 - G_1G_2H_2 + G_1G_2G_3H_3 + G_1G_2G_3G_4}$$

3.

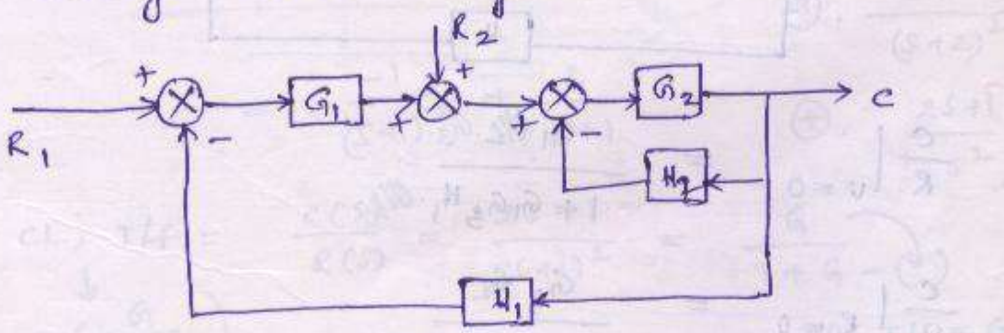


2 to 1 :-

$$\frac{G_2}{1 + G_3H_1 \cdot G_2} \quad \& \quad (G_3 \cdot G_4 + G_5)$$

$$= \frac{G_1 G_2 (G_3 G_4 + G_5)}{1 + G_3H_1G_2 + G_1G_2(G_3G_4 + G_5) \cdot H_2}$$

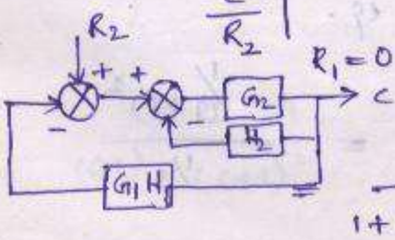
4. find the response due to R_1 and R_2 acting simultaneously.

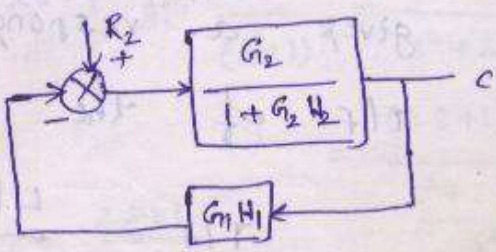


c due to R_1,

$$\left.\frac{C}{R_1}\right|_{R_2=0} = \frac{G_1G_2}{1 + G_2H_2 + G_1G_2H_1}$$

$$\left.\frac{C}{R_2}\right|_{R_1=0} =$$

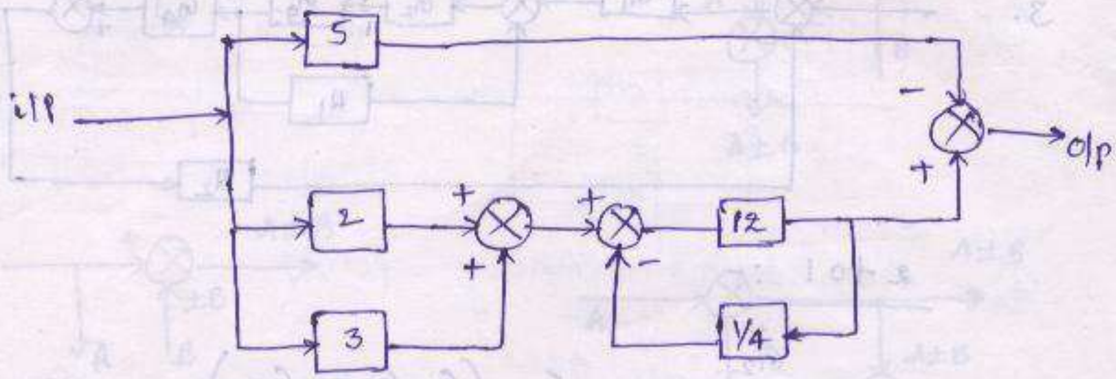




$$= \frac{G_2}{1 + G_2H_2 + G_2G_1H_1}$$

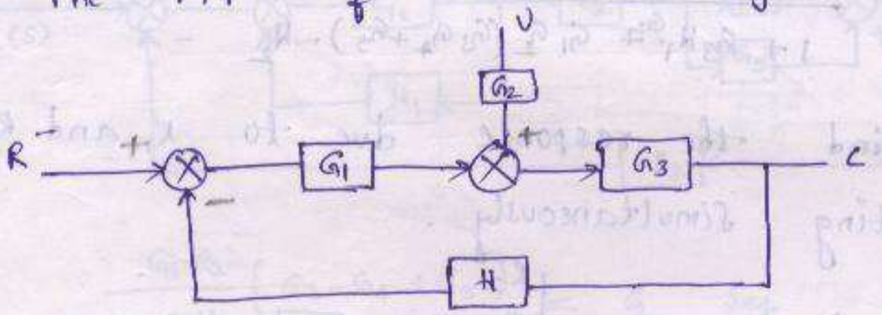
$$\therefore \quad C = \frac{G_1 G_2 R_1 + G_2 R_2}{1 + G_2 H_2 + G_1 G_2 H_1}$$

5. find the gain of the system,



Ans: 10.

6. The T/f of the block diagram below is



$$\left.\frac{C}{R}\right|_{U=0} = \frac{G_1 \cancel{G_2} \cdot G_3}{1 + G_1 G_3 H_1 \cdot \cancel{G_2}}$$

$$\left.\frac{C}{U}\right|_{R=0} = \frac{G_3 G_2}{1 + G_1 G_3 H_1 \cdot \cancel{G_2}}$$

7. A linear time invariant system initially at rest, when subjected to unit step, gives a response of $y = t e^{-t}$. The T/f of the system is.

$$T/f = \frac{L[U.S.R]}{L[U.S]} = \frac{1/(s+1)^2}{1/s}$$

8. The impulse response of an initially relaxed linear system is $e^{-2t}u(t)$ to produce a response of $te^{-2t}.u(t)$ the i/p must be equal to — ?

$$G(s) = \frac{C(s)}{R(s)}$$

$$\Rightarrow R(s) = \frac{C(s)}{G(s)} = \frac{1/(s+2)^2}{1/(s+2)}$$

$$= \frac{1}{s+2} = e^{-2t}.u(t).$$

9. The unit impulse response of an unity (H = 1) f/b control system is $c(t) = (-te^{-t} + 2e^{-t}).u(t), t \geq 0$
The open loop T/f (G) = ?

① $\frac{2s+1}{(s+1)^2}$

② $\frac{s+1}{s^2}$

③ $\frac{s+1}{(s+2)^2}$

④ $\frac{2s+1}{s^2}$

CL. T/f = L[impulse response] with initial condi = 0

$$= \frac{-1}{(s+1)^2} + \frac{2}{s+1}$$

$$\text{CL, T/f} = \frac{C(s)}{R(s)} = \frac{2s+1}{(s+1)^2} = \frac{G}{1+G-(\,)}$$

$$\left(\frac{G}{1+GH}\right)$$

O/L T/f = G

At H = 1, $\frac{G}{1+G}$

$$= \frac{2s+1}{(s+1)^2 - (\,)}$$

$$\text{OL T/f (G)} = \frac{2s+1}{(s+1)^2 - 2s - 1}$$

$$\frac{2s+1}{(s+1)^2} = \frac{2s+1}{s^2+2s+1}$$

$$= \frac{2s+1}{s^2}$$

$$= \frac{(2s+1)/s^2}{\frac{s^2+2s+1}{s^2}}$$

(or) $\frac{2s+1}{(s+1)^2} = \frac{G}{1+G} \Rightarrow G = \frac{2s+1}{s^2}$

10. find OL DC gain of a unity f/b system of CL T/f is $\frac{s+4}{s^2+7s+13}$

① 4/13
② 4/9
③ 4
④ 14

DC gain means $f=0$, ie sub. $s=0$,

$s = j\omega = j2\pi f$

$\longrightarrow \frac{4}{13}$

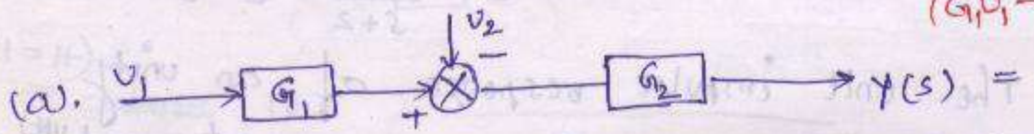
$/G$ OL gain

$\frac{G}{1+G-G}$

11. In block diagram shown the o/p $Y(s) = ?$

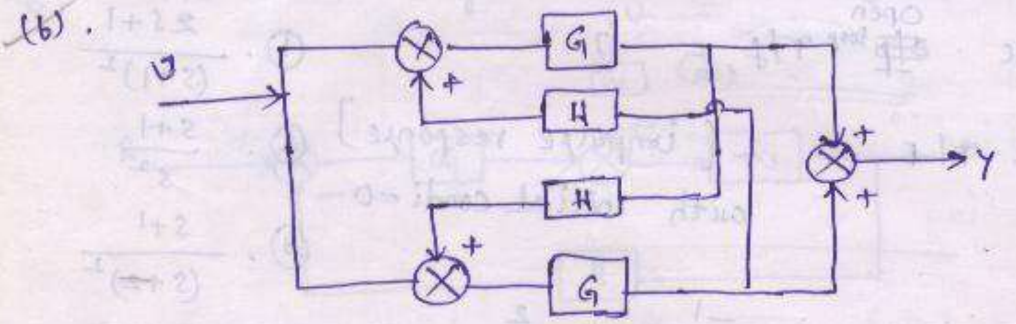
$(G_1U_1 - U_2)G_2$

(a).



$= G_1G_2U_1 - G_2U_2$

(b).



$$Y(s) = \frac{G}{1-GH} + \frac{G}{1-GH} = \frac{2G}{1-GH}$$

⟹ Signal flow Graph:-

It is a graphical representation of the system b/w the set of linear algebraic eq.s.

DE
LT
LAE
linear Algebraic equation
TF
SFG
TF

Q. construct signal flow graph for the following LAE s.

a). $x_2 = Ax_1$ b) $x_5 = x_1 - 2x_2 + 3x_3 + 10x_4$

c). $x_2 = 7x_1$, $x_3 = 5x_1$, $x_4 = 3x_1$

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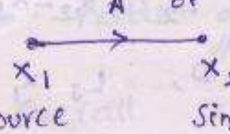
d). $y = mx + c$.

o/p (y), i/p (x)

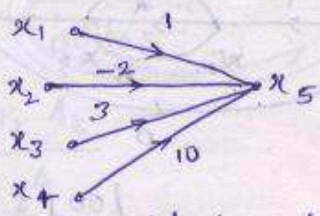
Ans :-

A → path gain or transmittance

a).

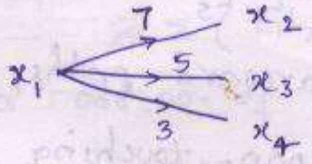


b).



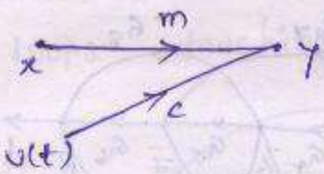
If all the signals are added at a particular node called as additional rule.

c).



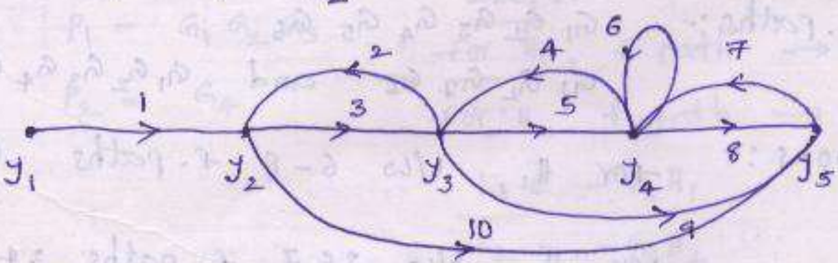
If the signal is transmitted from single node to many called transmission rule.

d).



Q. $y_2 = y_1 + 2y_3$, $y_3 = 3y_2 + 4y_4$, $y_4 = 5y_3 + 6y_4 + 7y_5$

$y_5 = 8y_4 + 9y_3 + 10y_2$



→ A node should be touched only once while selecting forward path/Loop.

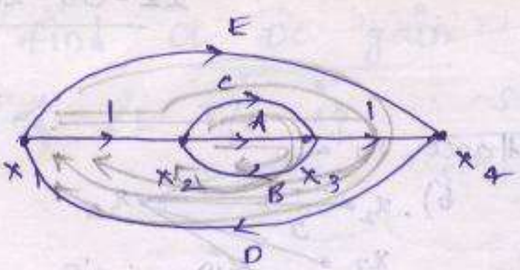
Loop :-

It is a path which terminates on the same node where it is started.

Non-touching loop :-

If there is no common node b/w 2 or more loop then it is called as non-touching loop.

Q.



f/w path B, $x_2 - x_3$

L_1 $x_2 - x_3$ (A)

L_2 $x_2 - x_3$ (C)

for D, L_3 $x_1 x_2 x_3 x_4$

L_4 $x_1 x_2 x_3 x_4$

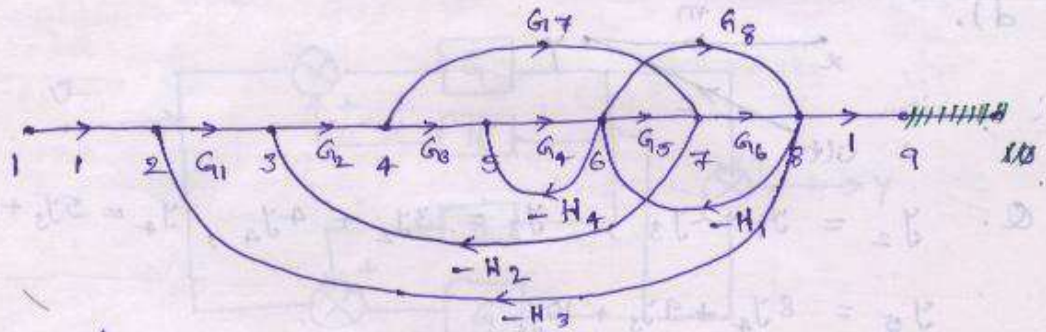
L_5 $x_1 x_4$

loops = 5

for Non-touching, $L_1 L_3$, L_4, L_5 | $L_2 L_3$, L_4, L_5

$= L_1 L_5$ & $L_2 L_5$

Q. find the no. of farward paths, loops, 2 non-touching and 3 non-touching loops.



f. paths :- $G_1 G_2 G_3 G_4 G_5 G_6$

$G_1 G_2 G_7 G_6$ and $G_1 G_2 G_3 G_4 G_8$

loops :- for H_1, b/w 6 - 8 f. paths $G_5 G_6$, G_8 } 2

for H_2, b/w 3 & 7, f. paths 34567, 347 } 2

for H_3, b/w 2 & 8, f. paths 2345678, 23478, 234568 } 3

for H_4, b/w 5 & 6, f. paths 56 } 1

Total no. of loops: 8

Two-non touching loops → 3

Three-non touching loops → 0

[To findout Three-non touching loop, first select two-non touching loops and then check with other].

Mason's Gain formula :-

- finding overall T/F
- ratio of any two nodes

$$\text{Overall } T/F = \sum_{k=1}^{i} \frac{P_k \Delta_k}{\Delta}$$

P_k – kth forward path gain

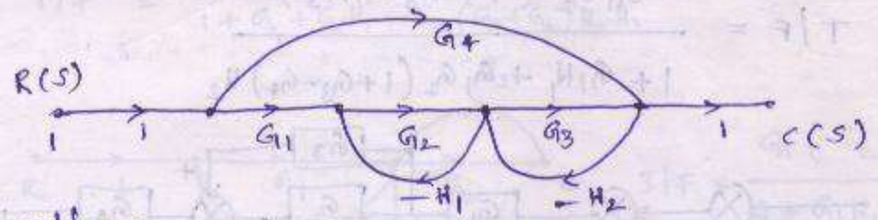
$$\Delta = 1 - \Sigma(L_1 + L_2 + L_3 + \dots) + \underbrace{\Sigma(L_1L_2 + L_1L_3 + \dots)}_{\text{two-non touching}}$$

$$\cancel{+} - \underbrace{\Sigma(L_1L_2L_3 + \dots)}_{\text{Three-non touching}} + \Sigma(L_1L_2L_3L_4 + \dots)$$

for odd no. of non-touching loops, take opposite sign for loop gain & for same sign for even.

Δ_k – is obtained from Δ, by removing the loops touching the k th forward path.

Q.



f. paths:

$P_1 = G_1 G_2 G_3$

$P_2 = G_4$

Loops:-

for H_1, f. path → 1

for H_2, f. path → 1

② loops

$L_1 = -G_2 H_1$

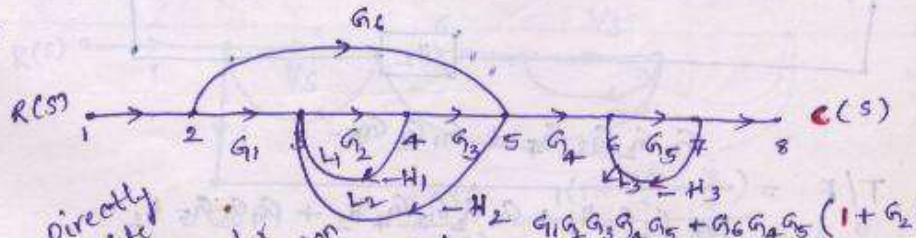
$L_2 = -G_3 H_2$

$$\Delta = 1 - (-G_2H_1 - G_3H_2)$$

$$\Delta_1 = 1 \quad ; \quad \Delta_2 = 1 - (-G_2H_1)$$

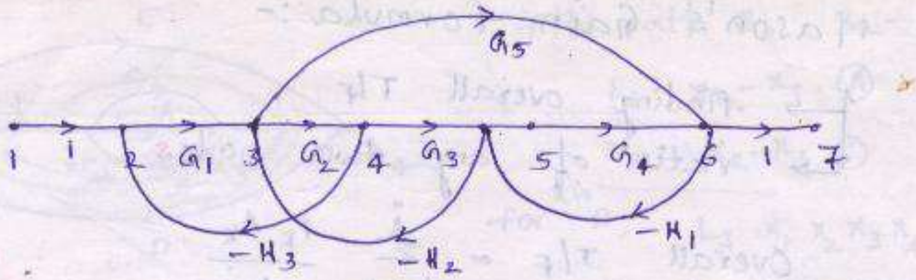
$$T/F = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_4(1 + G_2H_1)}{1 + G_2H_1 + G_3H_2}$$

Q.



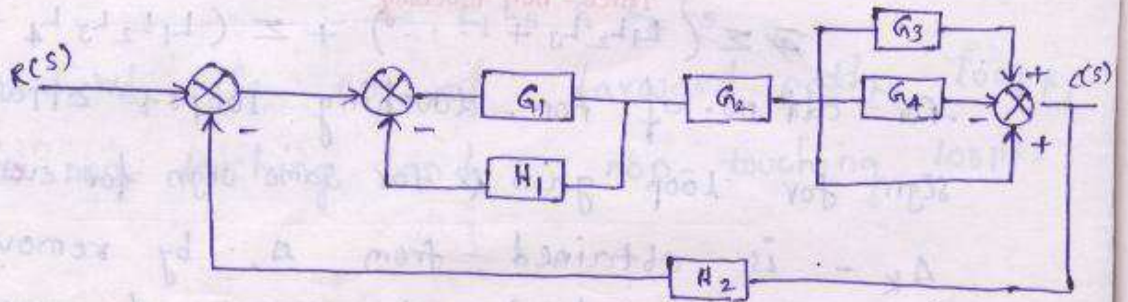
Directly write the Transfer function of → $T/F = \dfrac{G_1G_2G_3G_4G_5 + G_6G_4G_5(1 + G_2H_1)}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + L_1L_3 + L_2L_3}$

Q.



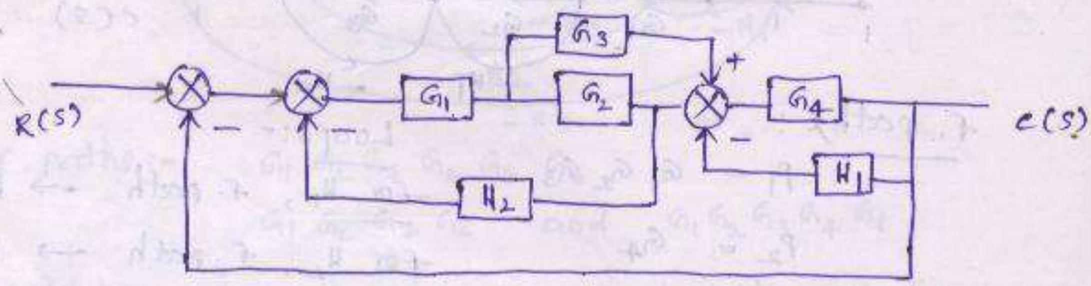
$$T/F = \frac{G_1G_2G_3G_4 + G_1G_5(1+0)}{1 + G_1G_2H_3 + G_2G_3H_2 + G_4H_1 + G_1G_2H_3G_4H_1 - G_5H_1H_2}$$

Q.



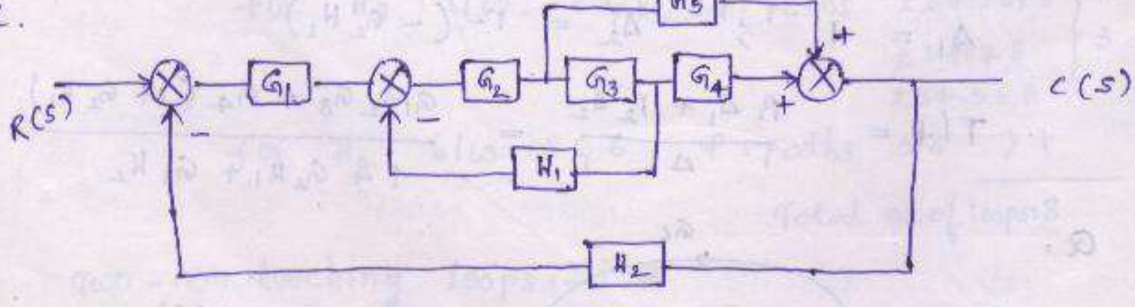
$$T/F = \frac{G_1G_2(1 - G_4 + G_3)}{1 + G_1H_1 + G_1G_2(1+G_3-G_4)H_2}$$

Q.



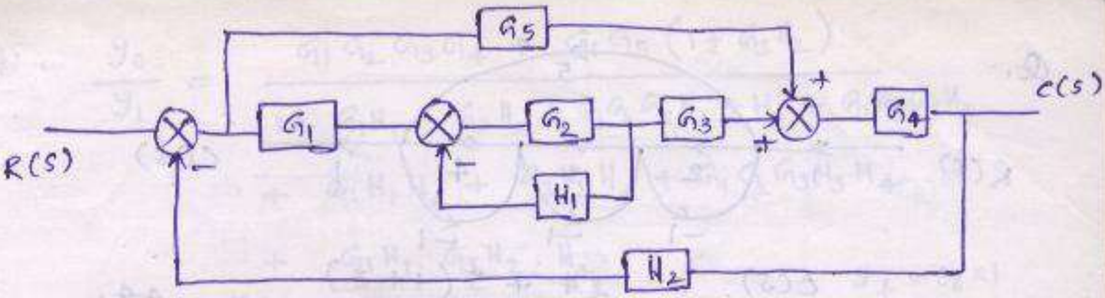
$$T/F = \frac{G_1G_2G_4 + G_1G_3G_4(1+0)}{1 + G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4 + G_1G_2H_2G_4H_1}$$

Q.



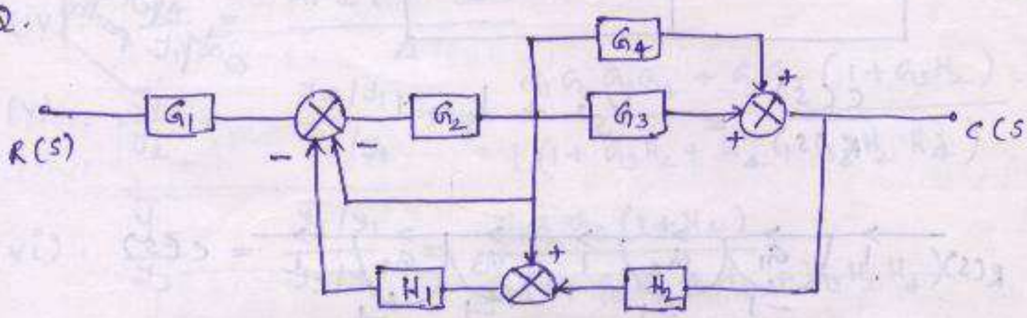
$$T/F = \frac{G_1G_2G_3G_4 + G_1G_2G_5}{1 + G_2G_3H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_5H_2}$$

Q.



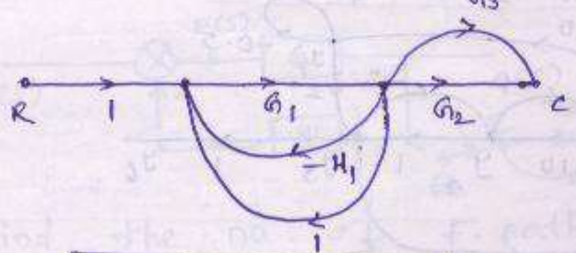
$$T/F = \frac{G_1G_2G_3G_4 + G_5G_4(1+G_2H_1)}{1+G_2H_1 + G_1G_2G_3G_4H_2 + G_5G_4H_2 + G_2H_1G_5G_4H_2}$$

Q.



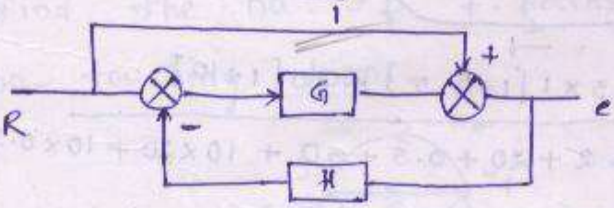
$$T/F = \frac{G_1G_2(G_3+G_4)}{1+G_2+G_2H_1+G_2(G_3+G_4)H_2H_1}$$

Q.



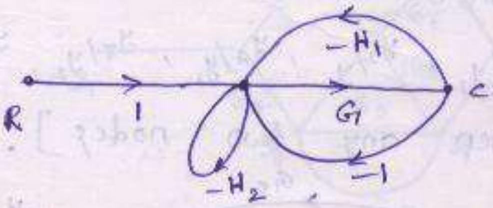
$$T/F = \frac{G_1(G_2+G_3)}{1+G_1H_1+(-G_1)}$$

Q.



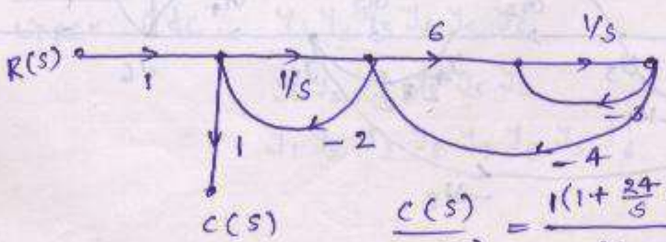
$$T/F = \frac{1+G}{1+GH}$$

Q.



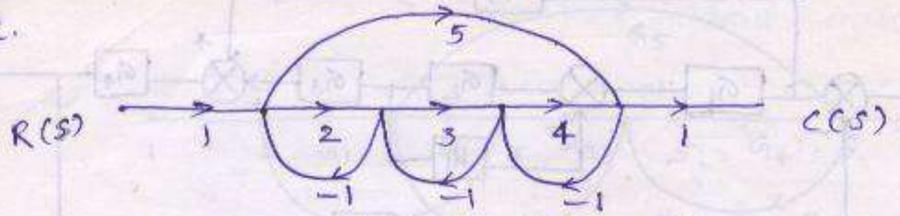
$$T/F = \frac{G}{1+H_2+G+GH_1}$$

Q.



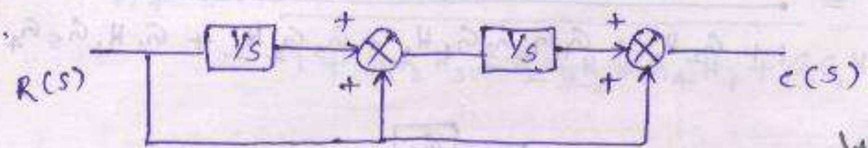
$$\frac{C(S)}{R(S)} = \frac{1\left(1+\frac{24}{s}+\frac{3}{s}\right)}{1+\frac{2}{s}+\frac{24}{s}+\frac{3}{s}+\frac{6}{s^2}}$$

Q.



$$\frac{C(S)}{R(S)} = \frac{24 + 5(1+3)}{1+2+3+4+5+8} = \frac{44}{23}$$

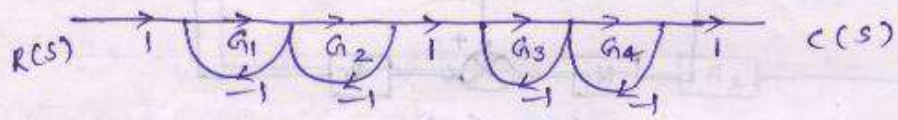
Q.



only forward paths

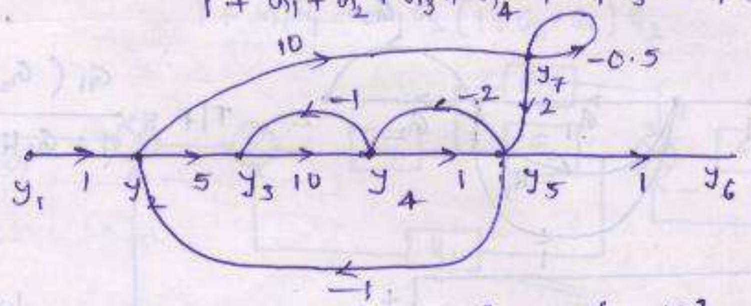
$$\frac{C(S)}{R(S)} = 1/S^2 + 1/S + 1$$

Q.



$$T/F = \frac{G_1 G_2 G_3 G_4}{1 + G_1 + G_2 + G_3 + G_4 + G_1G_3 + G_1G_4 + G_2G_3 + G_2G_4}$$

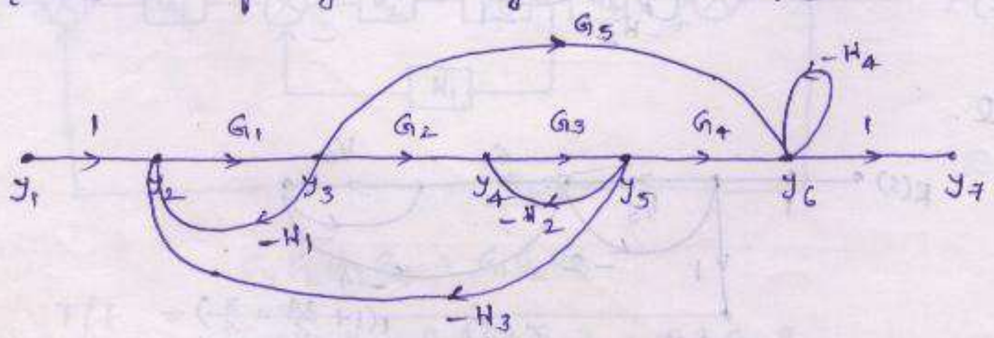
Q.



$$T/F = \frac{10\times5\times1\,[1+0.5] + 20\,[1+10]}{1+10+2+20+0.5+50+10\times20+10\times0.5 + 2\times0.5 + 50\times0.5} = 0.9$$

Q. Find y_6/y_1, y_7/y_1, y_2/y_1, y_4/y_1, y_7/y_2, y_5/y_3, y_4/y_3

[Ratio of given any two nodes].



(i). $$\frac{y_6}{y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1+G_1H_1+G_3H_2+G_1G_2G_3H_3+H_4+G_1H_1G_3H_2 + G_1H_1H_4+G_3H_2H_4+G_1G_2G_3H_3H_4 + G_1H_1.G_3H_2.H_4}$$

$y_7 = y_6 \times 1 = y_6$

(ii). $$\frac{y_7}{y_1} = \frac{y_6}{y_1}$$

$(1-(L_1+L_2)+L_1L_2)$

(iii). $$\frac{y_2}{y_1} = \frac{1(1+G_3H_2+H_4+G_3H_2H_4)}{\Delta}$$

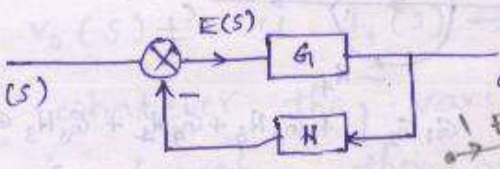
(iv). $$\frac{y_4}{y_1} = \frac{G_1G_2(1+H_4)}{\Delta}$$

(v). $$\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1(1+G_3H_2+H_4+G_3H_2H_4)}$$

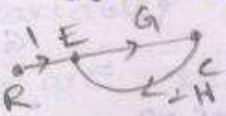
(vi). $$\frac{y_5}{y_3} = \frac{y_5/y_1}{y_3/y_1} = \frac{G_1G_2G_3(1+H_4)}{G_1(1+G_3H_2+H_4+G_3H_2H_4)}$$

(vii). $$\frac{y_4}{y_3} = \frac{y_4/y_1}{y_3/y_1} = \frac{G_1G_2(1+H_4)}{G_1(1+G_3H_2+H_4+G_3H_2H_4)}$$

Q.

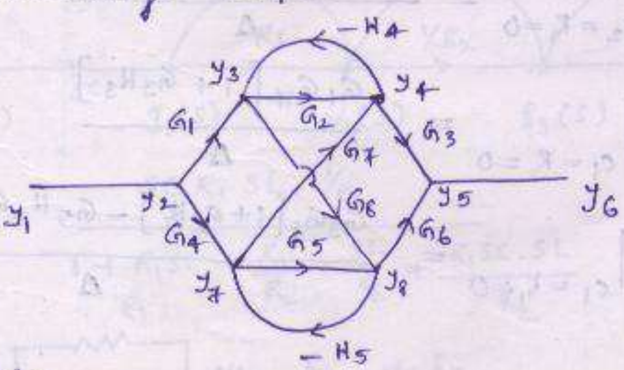


$\frac{E(S)}{R(S)} = ?$



Ans: $\frac{1}{1+GH}$

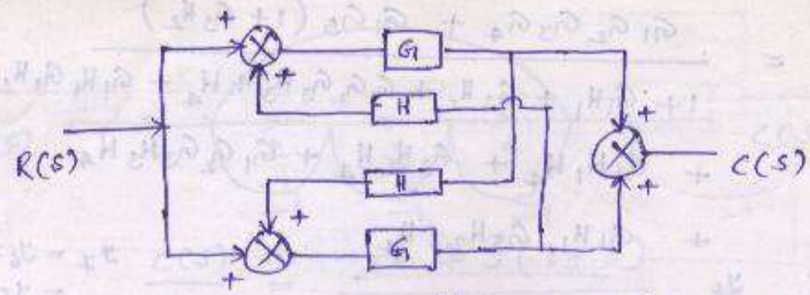
Q. find the no. of f. paths, loops and two non-touching loops.



f. paths:-

on upper side: $y_1y_2y_3y_4y_5y_6$
$y_1y_2y_3y_8y_5y_6$
$y_1y_2y_3y_8y_7y_4y_5y_6$ } 3

Q.

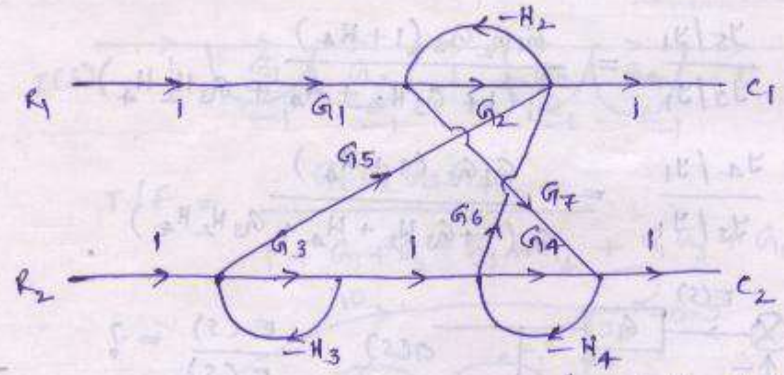


$$= \frac{2G}{1-GH}$$

$$T/f = \frac{G + G^2H_2 + G + G^2H}{1 - G^2H^2}$$

$$= \frac{2G\,[1+GH]}{(1+GH)(1-GH)} = \frac{2G}{1-GH}$$

Q. find C_1/R_1, C_1/R_2, C_2/R_1, C_2/R_2. [Multi i/p, Multi o/p]



(i). $\left.\frac{C_1}{R_1}\right|_{C_2=R_2=0} = \frac{G_1G_2\,[1+G_3H_3+G_4H_4+G_3H_3G_4H_4] - G_1G_7H_4G_6\,[1+G_3H_3]}{\Delta}$

(ii). $\left.\frac{C_1}{R_2}\right|_{C_2=R_1=0} = \frac{G_5\,[1+G_4H_4] + G_3G_6}{\Delta}$

(iii). $\left.\frac{C_2}{R_1}\right|_{C_1=R_2=0} = \frac{G_1G_7\,[1+G_3H_3]}{\Delta}$

(iv). $\left.\frac{C_2}{R_2}\right|_{C_1=R_1=0} = \frac{G_3G_4\,[1+G_2H_2] - G_5H_2G_7 - G_3G_6H_2G_7}{\Delta}$

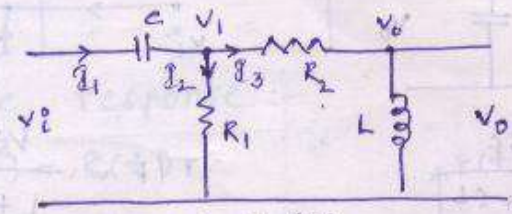
SFG's for Electrical N/ω :-

Ref: 1. Ogota 2. B.C. kuo

Steps :-

1. Select Branch current or node voltage
2. Apply L.T. to all the var's & system components.
3. write the eq's of V/I
4. construct SFG.

Eg :-



$$T/F = \frac{V_0}{V_i} = \frac{R_1 SL}{\frac{1}{SC}[R_1+R_2+SL]+R_1[R_2+SL]}$$

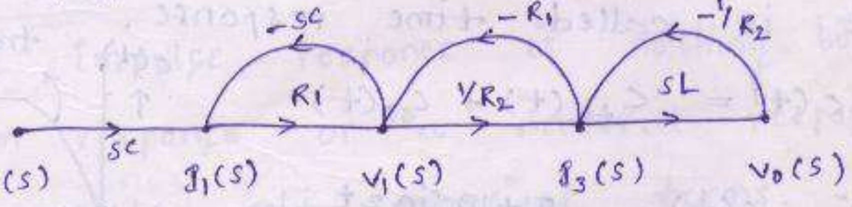
$$I_1(s) = \frac{V_i(s) - V_1(s)}{1/sc} = sc[V_i(s) - V_1(s)] \rightarrow ①$$

$$V_1(s) = I_2(s) \cdot R_1 = R_1[I_1(s) - I_3(s)] \rightarrow ②$$

$$I_3(s) = \frac{V_1(s) - V_0(s)}{R_2} \rightarrow ③$$

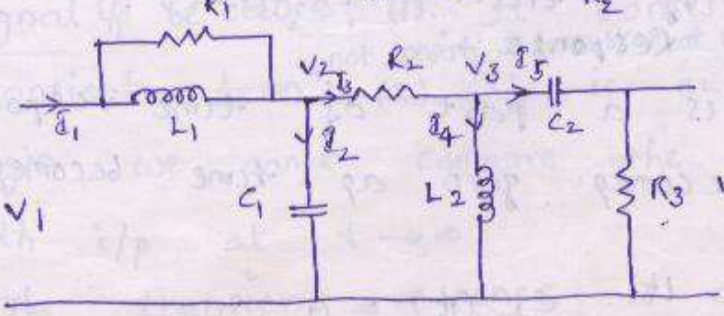
$$V_0(s) = SL \cdot I_3(s) \rightarrow ④$$

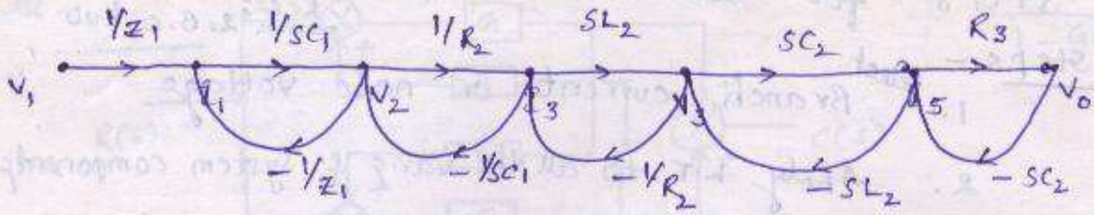
→ whatever the variables along the series branch, they are taken as nodes in SFG.



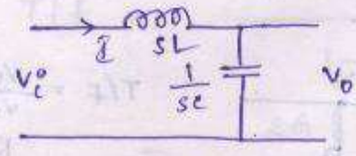
$$\frac{V_0(s)}{V_i(s)} = T/F = \frac{SC R_1 SL \cdot \frac{1}{R_2}}{1 + R_1 SC + \frac{R_1}{R_2} + \frac{SL}{R_2} + \frac{R_1 SC \cdot SL}{R_2}} =$$

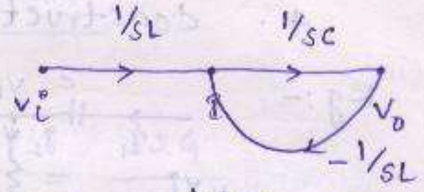
Q.





Q. Draw SFG for,





$$T/F = \frac{1/sC}{1/sC + sL} = \frac{1}{1+s^2LC}$$

$$T/F = \frac{1/s^2LC}{1 + 1/s^2LC} = \frac{1}{1+s^2LC}$$

→ TIME DOMAIN ANALYSIS :-

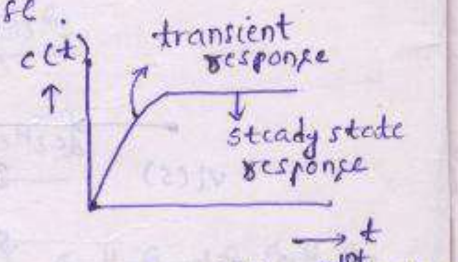
Ref: 1. Nagrath / Gopal

- time domain specifications
- e_{ss}
- Responses

→ Time Response :-

If the response of the system varies w.r.t time then it is called time response.

Time Response $c(t) = c_{tr}(t) + c_{ss}(t)$



Ex:- $c(t) = 5 + 10\sin 2t + e^{-10t}\cos 5t + \ldots$

$c_{tr}(t) = e^{-10t}\cos 5t + \ldots$

Identify $c_{tr}(t)$ and $c_{ss}(t)$.

$c_{ss}(t) = 5 + 10\sin 2t$

→ Transient Response :-

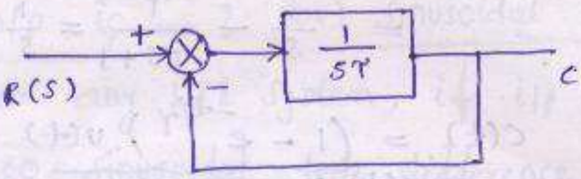
It is a part of time response that becomes zero as time becomes very large.

$$\lim_{t\to\infty} c_{tr}(t) = 0$$

→ Steady state Response :-

It is a part of time response that remains after the transients (or (becomes zero)) died out.

→ Time response for the I order systems :-



C/L T/F :

$$\frac{C(S)}{R(S)} = \frac{1}{\tau S+1}$$

↳ 1. Impulse response :-

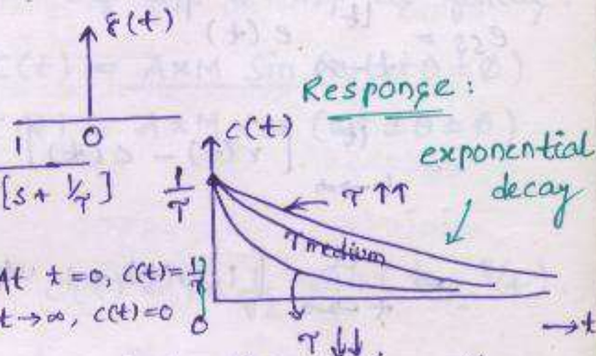
$r(t) = \delta(t)$

$R(S) = 1$

$$C(S) = \frac{1}{\tau S+1} = \frac{1}{\tau[S+1/\tau]}$$

$$\Rightarrow C(t) = \frac{1}{\tau} \cdot e^{-t/\tau}$$

$\left\{ \begin{array}{l} \text{At } t=0, \ C(t)=\frac{1}{\tau} \\ t\to\infty, \ C(t)=0 \end{array} \right.$



* Error is nothing but deviation of the o/p from the ref. i/p.

$e(t) = r(t) - c(t)$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} [r(t) - c(t)]$$

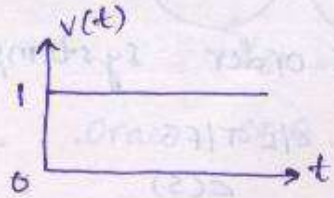
* The impulse response is nothing but a system response or a natural response. It consists only transient terms.

* The e_{ss} are not defined for impulse signal i/p because, (1). It consists only the transient term & not consists ss term b'coz at the ss, there is no i/p exists. (2). I/p is existed only at origin, we can't compare the response with i/p at $t\to\infty$

* The transient response is only due to system time constant and ss response is

only due to i/p.

Unit step input :-



$r(t) = 1 \cdot u(t)$

$R(s) = \frac{1}{s}$

$C(s) = \frac{1}{s(\tau s+1)}$

$= \frac{1}{s} - \frac{\tau}{\tau s+1} = \frac{1}{s} - \frac{1}{s+1/\tau}$

$c(t) = \underbrace{(1}_{ss} - \underbrace{e^{-t/\tau}}_{\text{Tr. response}}) \cdot u(t)$

Response

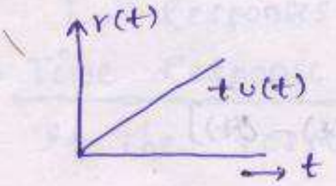
$e_{ss} = \lim_{t\to\infty} e(t)$

$= \lim_{t\to\infty} [r(t) - c(t)]$

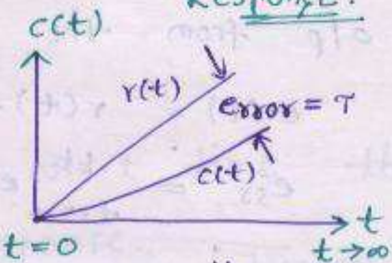
$= \lim_{t\to\infty} \left[1 \cdot u(t) - 1 \cdot u(t) + e^{-t/\tau} \cdot u(t)\right]$

$= 0$

Unit ramp input :-



Response:



$r(t) = t\,u(t)$

$R(s) = 1/s^2$

$C(s) = \frac{1}{s^2(\tau s+1)}$

$= \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{\tau s+1}$

$= (\underbrace{t - \tau}_{S.S.} + \underbrace{\tau \cdot e^{-t/\tau}}_{Tr.})\, u(t)$

ss. error $= \lim_{t\to\infty} e(t)$

$= \lim_{t\to\infty} \left[t\,u(t) - t\,u(t) + \tau u(t) - \tau e^{-t/\tau} u(t)\right]$

$= +\tau$

Unit Parabolic input :-

$r(t) = 1 \cdot \frac{t^2}{2} u(t)$

$R(s) = 1/s^3$

$C(s) = \frac{1}{s^3(\tau s+1)}$

purchase ! R.K. Kanodia

↳ Sinusoidal Response :

Q. The CL T/f of an unity f/b system is given by $\frac{C(s)}{R(s)} = \frac{1}{s+1}$. for the i/p $r(t) = \sin t$, the SS. o/p is — ? (or) Sinusoidal response is — ?

* for any LTI system, if i/p is sinusoidal, the o/p also sinusoidal but difference in magnitude & phase shift. The standard form of i/p & o/p as follows.

$r(t) = A\sin(\omega t \pm \theta) \Rightarrow c(t) = A \times M \sin(\omega t \pm \theta \pm \phi)$

$r(t) = A\cos(\omega t \pm \theta) \rightarrow c(t) = A \times M \cos(\omega t \pm \theta \pm \phi)$.

$r(t) = \sin t$, $\Rightarrow \omega = 1$

Replace $s = j\omega = j$.

$\Rightarrow \frac{C(s)}{R(s)} = \frac{1}{j+1}$ $\therefore c(t) = 1 \times \frac{1}{\sqrt{2}} \sin(t - \pi/4)$.

$\therefore M = \frac{1}{\sqrt{2}}$

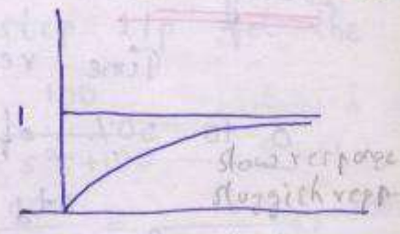
$\angle\phi = \frac{\angle 1}{\angle(j+1)} = \frac{0^\circ}{45^\circ} = -45^\circ$

Case 2 :- $\xi = 1$:-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\Rightarrow C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

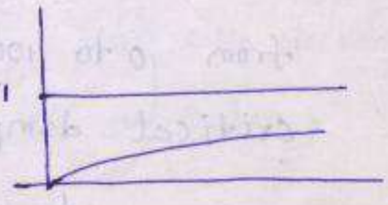
$$\Rightarrow c(t) = 1 - \cancel{\omega_n t \cdot e^{-\omega_n t}} - e^{-\omega_n t}$$



Case 3 :- $\xi > 1$:-

$$C(s) = \frac{\omega_n^2}{s(s+P_1)(s+P_2)}$$

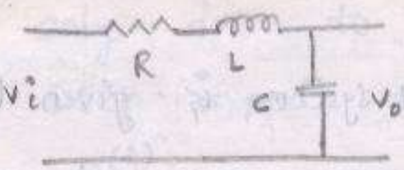
$$c(t) = 1 - k_1 e^{-P_1 t} - \cancel{k_2 e^{-P_2 t}}$$



ξ - damping ratio

$\xi\omega_n$ - actual damping

damping factor



$$\frac{V_0(s)}{V_i(s)} = \frac{1/sC}{R+sL+1/sC} = \frac{1}{s^2LC+sCR+1}$$

$$= \frac{1/LC}{s^2+\frac{sR}{L}+\frac{1}{LC}}$$

$$\omega_n = 1/\sqrt{LC}$$

$$2\xi\omega_n = \frac{R}{L} \Rightarrow \xi = \frac{R}{2L}\cdot\sqrt{LC}$$

$$\rightarrow \boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

$$\left(R = 2\xi\sqrt{L/C}\right)$$

TF $\frac{1}{1+sT}$

OL BW $= 1/T$

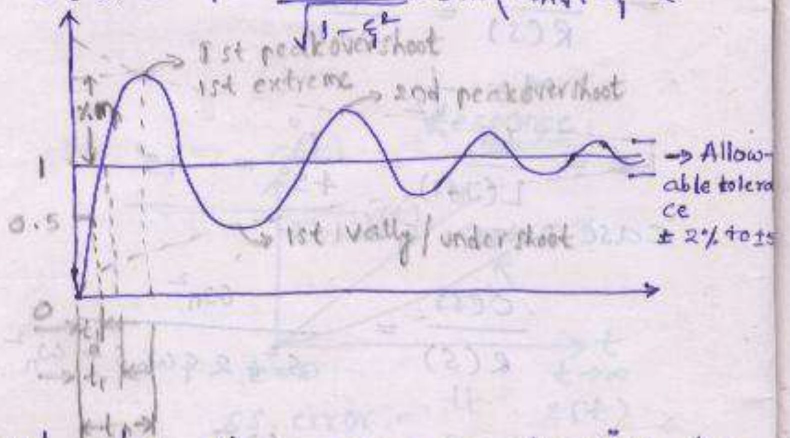
CL BW $= \frac{1+k}{T}$ → system gain

Time Domain Specifications:-

For the time domain specifications consider the underdamped system because the rise time and settling time is minimum. For $0<\xi<1$, the unit step response of the system is

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

(or) $\cos^{-1}\xi$



*** Delay time :-**

Time required for the response to rise from 0 to 50% of the final value.

$$t_d = \frac{1+0.7\xi}{\omega_n}\ \text{sec}$$

*** Rise time :-**

Time required for the response to rise from 0 to 100% for underdamped, 5 to 95% for critical damped, 10 to 90% for overdamped.

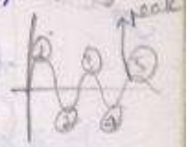
$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d}\ \text{radians} = \frac{\pi - \cos^{-1}\xi}{\omega_d}\ \text{sec}$$

*** Peak time :-**

Time required for the response to rise and reach the peaks of the response.

$$t_p = \frac{n\pi}{\omega_d} \quad (\text{for 1st peak } n=1)$$

3rd peak



$$= \frac{\pi}{\omega_d} \qquad 3^{rd} \text{ peak, } t_p = \frac{5\pi}{\omega_d}$$

$$1^{st} \text{ vally } t_p = \frac{2\pi}{\omega_d}$$

* **Settling time :-**

Time required to rise and stay within the specified tolerance band $\% \pm 2$ or $\pm 5\%$.

$$t_s = 4\tau = \frac{4}{\xi\omega_n} \rightarrow \pm 2\%$$

$$= 3\tau = \frac{3}{\xi\omega_n} \rightarrow \pm 5\%$$

These values are also valid for overdamped and critical damped.

* **Peak overshoot :-**

It indicates normalized difference b/w s.s. o/p to 1st peak of the time response.

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$= (c(t_p) - 1) \times 100$$

$$= e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

Q. Find the $\% M_p$ for unit step i/p for the given function (i). $\frac{C(s)}{R(s)} = \frac{100}{s^2+100}$ → undamped $\xi = 0$

(ii). $\frac{C(s)}{R(s)} = \frac{16}{s^2+8s+16}$ ($\xi = 1$, critical) (iii). $\frac{C(s)}{R(s)} = \frac{16}{s^2+100s+16}$ so $\% M_p = 100\%$

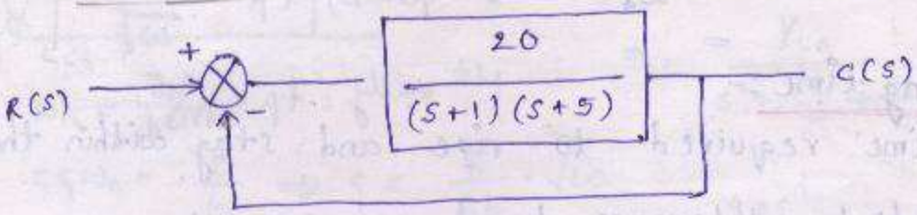
(i). critical damped → $\% M_p = 0\%$

As ξ increases from 0 to 1, the $\% M_p$ decreases

$\xi > 1$, the system does not have the oscillations

hence no $\% M_p$ and no peak time.

Q. A block diagram is shown in fig. The time period of oscillations before reaching the SS, is — ?



$$T_{oscillations} = \frac{2\pi}{\omega_d} \qquad \frac{20}{s^2+6s+25}$$

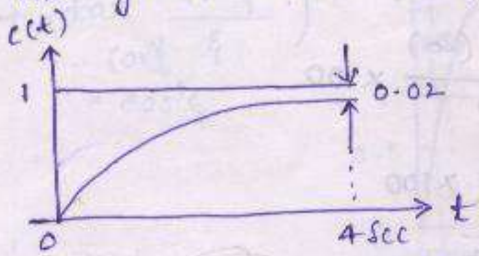
So $\omega_d = \omega_n\sqrt{1-\xi^2}$

$= 4.$ $\Rightarrow \omega_n = 5,\ \xi = 0.6$

Q. find no. of oscillations (or) cycles

$$N = \frac{t_s}{T_{osci}}$$

Q. find the time const. of the system for the given unit step response.



$t_s = 4T$

$4 = 4T$

$\Rightarrow T = 1\ sec$

Q. Given $G(s) = \frac{25}{s(s+4)}$, $H(s) = 1$. find the time domain specifications.

find unit step response for above system.

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+4s+25}$$

$\omega_n = 5$ rad/sec

$\xi = 0.4$, $\omega_d = \omega_n\sqrt{1-\xi^2} = 4.5$ rad/sec

$$t_d = \frac{1+0.7\xi}{\omega_n} = 0.256\ sec$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} = 0.44\ sec$$ ← (radians)

$$t_p = \frac{\pi}{\omega_d} = 0.69\ sec$$

$\pm 2\%\ t_s = 2\ sec$ → $k=1$, unit step i/p,

$\pm 5\%\ t_s = 1.5\ sec$

(b) $c(t) = 1 - \dfrac{e^{-0.4\times5t}}{\sqrt{1-0.4^2}} \cdot \sin(4.5t + \cos^{-1}0.4)$

unit step response — steady state value, it never effect the transient value. Exponent: depends on system gain & magnitude of system i/p. k

Q. for a system shown in fig. find the time domain specifications when the unit step i/p is applied. find unit step response for above system.

$$\frac{C(s)}{R(s)} = \frac{20}{s^2+5s+24} = \frac{20}{24} \cdot \frac{24}{s^2+5s+24}$$

steady state value

$\omega_n = 4.89\ rad/sec$

$\xi = 0.51,\quad \omega_d = 4.2\ rad/sec$

$t_d = 0.277\ sec$ $\qquad \pm 2\%\ t_s = 1.6\ sec$

$t_r = 0.5\ sec$ $\qquad \%\ M_p = 15.43\%$

$t_p = 0.74\ sec$ $\qquad c(t) = \dfrac{20}{24}\left(1 - \dfrac{e^{-2.5t}}{0.859} \cdot \sin(4.2t + 1.03)\right)$

* As ξ increases, the poles moves towards the L.H.S and nearer to the real axis. Hence the frequency of osci. are decreases.

As the freq. of osci decreases, the time specifications t_d, t_r, t_p must be increases.

As ξ increases the $\%\ M_p$ must be decreases.

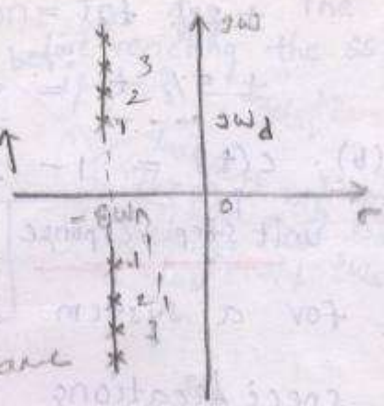
As ξ increases the time const. should be decreases hence the settling time must be decreases. & BW ↓

$\omega_d \uparrow\uparrow$ $\quad$ $\xi\omega_n$ const.

$t_d, t_r, t_p \downarrow\downarrow$ $\quad$ $\omega_n\sqrt{1-\xi^2}\uparrow$

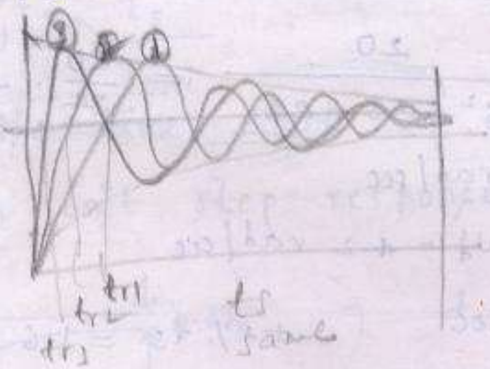
so $\xi\omega_n$ – const $\Rightarrow \sqrt{\omega_n^2 - \xi^2\omega_n^2}\uparrow$

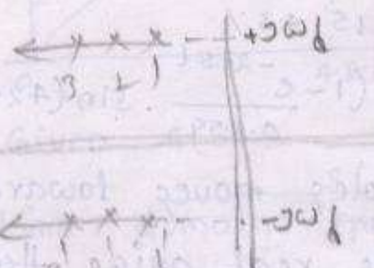
$\xi\omega_n$ const, so ω_n must be decreased; $\omega_n^2 \uparrow$; $\xi^2\omega_n^2$ const



$\xi\downarrow \Rightarrow \%M_p\uparrow$ $\quad$ t_s – same

→ * As poles moves vertically $\|^{el}$ to $j\omega$ axis, the damping factor is decreased and hence $\% M_p$ increases.





ω_d const ; $t_p = \frac{\pi}{\omega_d}$ → const.

$\simeq t_r, t_d$

$\omega_d = \sqrt{\omega_n^2 - \omega_n^2\xi^2}$, ξ increases as well as ω_n increases

$t_d = \frac{1+0.7\xi\uparrow}{\omega_n\uparrow}$ (approximately const) slight variation

$t_r = \frac{\pi - \cos^{-1}(\xi\uparrow)\downarrow}{\omega_d\uparrow}$ slightly

$\tau\downarrow\downarrow \rightarrow t_s\downarrow\downarrow$

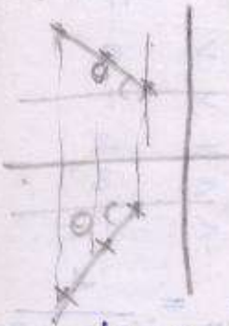
$\xi\uparrow\uparrow \rightarrow \% M_p \downarrow\downarrow$

* As ω_d is constant, the t_p is same.

Even t_r, t_d are approximately constant.

As the poles moves towards L.H.S., the time constant decreases hence t_s decreases.

As ξ increases, the % M_p decreases.



$\omega_d \uparrow\uparrow \rightarrow t_d, t_r, t_p \downarrow\downarrow$

$\tau \downarrow \rightarrow t_s \downarrow\downarrow$

$\xi = \cos\theta \rightarrow$ % $M_p \rightarrow$ const

* As the inclination of the poles is constant, the ξ is constant. hence the % M_p is constant.

Q. find the time domain specifications for unit step i/p. for the given system.

$$\frac{d^2y}{dt^2} + 4\cdot\frac{dy}{dt} + 8y = 8x$$

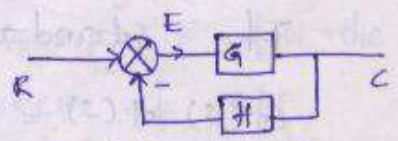
Ans:- $$\frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$$

steady state errors:-

It is the deviation of o/p from the reference i/p at the steady state [$t\to\infty$]

* $$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{s\cdot R(s)}{1+G(s)\cdot H(s)}$$



$$\frac{E(s)}{R(s)} = \frac{1}{1+GH}$$

* The SSE are depends on

(1). type of i/p (ie) $R(s)$ (2). type of system ie $G(s)H(s)$

Type of i/p :- $(R(s))$:- order. for step i/p $\rightarrow A.t^{0}U(t)$

$\rightarrow$ step i/p :- $r(t) = A\ u(t) \Rightarrow R(s) = \frac{A}{s}$

$$e_{ss} = \lim_{s\to 0} s . \frac{A/s}{1+G(s).H(s)} = \frac{A}{1+\lim_{s\to 0} G(s).H(s)} = \frac{A}{1+k_p}$$

k_p = static position error const

$$= \lim_{s\to 0} G(s).H(s) \Rightarrow k_p$$

$$\therefore e_{ss} = \frac{A}{1+k_p}$$

$\rightarrow$ Ramp i/p :-

$r(t) = At\ u(t) \Rightarrow R(s) = A/s^2$

$$\therefore e_{ss} = \frac{A}{k_v} \quad \because e_{ss} = \lim_{s\to 0} s . \frac{A/s^2}{1+G(s)H(s)} = \frac{A}{\underbrace{\lim_{s\to 0} s\,G(s)H(s)}_{k_v}}$$

$\rightarrow$ Parabolic i/p :-

$r(t) = At^2/2\ u(t) \Rightarrow R(s) = A/s^3$

$$e_{ss} = \lim_{s\to 0} \frac{s . A/s^3}{1+G(s).H(s)} \simeq \frac{A}{k_a} = \frac{A}{\lim_{s\to 0} s^2 (G(s).H(s))}$$

Type of systems :- System is represented as $G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\dots}{s^n(1+sT_a)(1+sT_b)\dots}$

* Type is nothing but no. of poles at origin.

* Order is nothing but total no. of poles in s-plane.

Type-n System

Type = i/p $\Rightarrow$ e_{ss} constant

$\rightarrow$ $\boxed{\text{Type} > \text{i/p} \Rightarrow e_{ss}\ 0}$

Type < i/p $\Rightarrow$ $e_{ss}\ \infty$

The standard form of the system is

$$G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\dots}{\underbrace{s^n}_{\text{Type}}(1+sT_a)(1+sT_b)\dots}$$

Type		i/p ① (step)(A)	e_{ss}
0	=	0	$\frac{A}{1+k}$ const.
1	>	0	0
2	>	0	0
3	⋮	0	⋮
4	⋮	0	⋮
⋮		⋮	

Type		i/p ① Ramp(At)	e_{ss}
0	<	1	∞
1	=	1	$\frac{A}{K}$ const
2	>	1	0
⋮	⋮	⋮	⋮

$K_p = \lim_{s \to 0} GH$

$K_v = \lim_{s \to 0} sGH$

$K_a = \lim_{s \to 0} s^2 GH$

Type		i/p ② parabolic($\frac{At^2}{2}$)	e_{ss}
0	<	2	∞
1	<	2	∞
2	=	2	A/K (const)
3	>	2	0
4	>	2	0
⋮	⋮	⋮	⋮

Q. Given $G(s) = \frac{10(s+2)}{s^2 (s+4)(s+10)}$. find the e_{ss} for the i/p $r(t) = 1 + 4t + t^2/2$, $H(s) = 1$.

Ans:-

Type		i/p	e_{ss}
2	>	0	0 +
2	>	1	0 +
2	=	2	$A/K = \frac{1}{20/40} = 2$

Q. Given $G(s).H(s) = \frac{10}{s(s+2)}$. find the e_{ss} for the following i/ps (1). $4t\,u(t)$ (2). $t^2 u(t)$ (3). $2u(t)$ (4). $(1+t+t^2)u(t)$

Ans:-

Type	i/p	e_{ss}
1		

$4t \to \frac{A}{K} = \frac{4}{10/2}$

$t^2 \to \infty$

$2u(t) \to 0$

$1+t+t^2 \to \infty$

$0 + K + \infty$

Q. find the e_{ss}, for unit ramp i/p for the given unity f/b control system of T/f

$$\frac{10}{s^3 + 20s^2 + 10} = \frac{C(s)}{R(s)}$$ (e_{ss} valid for CL stable system)

Ans:- The given T/f for closed loop is unstable hence the e_{ss} are not valid. ('s' term is missing)

Q. $\dfrac{C(s)}{R(s)} = \dfrac{10s + 10}{s^3 + 20s^2 + 10s + 10}$

→ e_{ss} are calculated to CL stable system, by using open loop (OL) T/F.

Ans:- OL T/F $= \dfrac{10s + 10}{s^3 + 20s^2 + 10s + 10 - 10s - 10}$

$= \dfrac{10s + 10}{s^3 + 20s^2} = \dfrac{10s + 10}{s^2(s + 20)}$

Type	i/p	e_{ss}
2	1	0

Q. Given $G(s) = \dfrac{k(s+2)}{s(s^3 + 7s^2 + 12s)}$, $H(s) = 1$. find The e_{ss} for the i/p $\frac{R}{2}t^2$.

Ans:- $G(s) = \dfrac{k(s+2)}{s^2(s^2 + 7s + 12)}$

$e_{ss} = \dfrac{A}{k} = \dfrac{R}{2k/12} = \dfrac{6R}{k}$.

Q. The OL T/F of the system is $\dfrac{k}{s(s+1)(s+2)}$ Determine the value of k, show that $e_{ss} = 0.1$ for unit ramp i/p.

Ans:- $e_{ss} = \dfrac{A}{k} = 1/_{k/2} = 0.1$

$\Rightarrow k = 20$

Q. find the e_{ss} for OL T/F of a unity f/b control system $G(s) = \dfrac{1}{(s+10)(s+20)}$ for the following i/ps (1). $10\,u(t)$ (2) $10t\,u(t)$ (3). $(10 + 10t + 10t^2)\,u(t)$

(1) → $\dfrac{A}{1+k} = \dfrac{10}{1 + 1/200}$

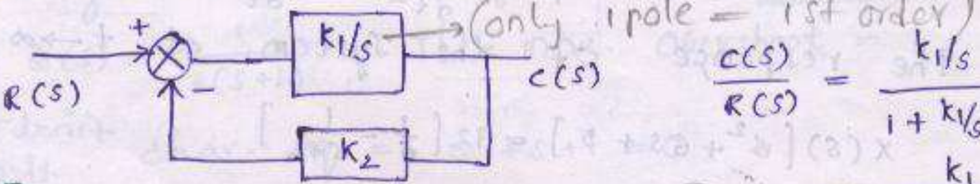
(2) → ∞

(3) → ∞

Ans:-

for $G(s) = \dfrac{(s+1)}{s^2(s+10)(s+20)}$ i/p's
(1) $10\,u(t) \rightarrow 0$
(2) $10t\,u(t) \rightarrow 0$
(3) $10t^2 u(t) \rightarrow \dfrac{A}{k} = \dfrac{20}{1/200}$
(4) $10 + 10t + 10t^2 \rightarrow$

Q. for the following system, the ss gain = 2, $\tau = 0.4$ sec, the values of k_1 and k_2 are



$$\frac{C(S)}{R(S)} = \frac{k_1/s}{1 + \frac{k_1}{s}\cdot k_2} = \frac{k_1}{s + k_1 k_2} = \frac{1/k_2}{1 + s\left[\frac{1}{k_1 k_2}\right]}$$

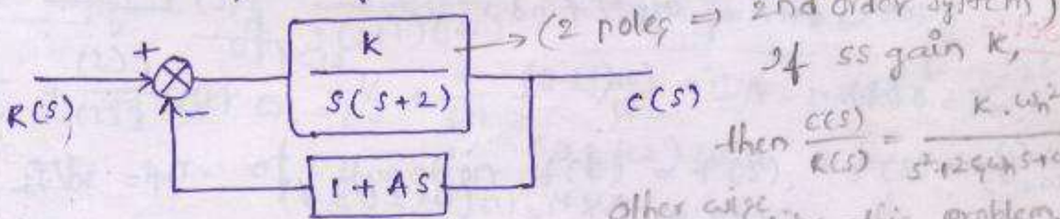
Ans:-

standard form $\frac{C(S)}{R(S)} = \frac{K}{1 + s\tau}$ 1st order

$\therefore \quad 2 = \frac{1}{k_2} \Rightarrow k_2 = 0.5$ {ss gain ~~K~~ = K = 2}

$0.4 = \frac{1}{k_1 k_2} \Rightarrow k_1 = 5$

Q. for the system shown in fig. with $\xi = 0.7$ and undamped natural freq. $\omega_n = 4$ rad/sec. The values of k and A are −?



If ss gain k, then $\frac{C(S)}{R(S)} = \frac{K\cdot\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

other wise like this problem,

char. equation : $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

(1+GH=0)

$$1 + \frac{k}{s(s+2)}\cdot(1 + As) = 0$$

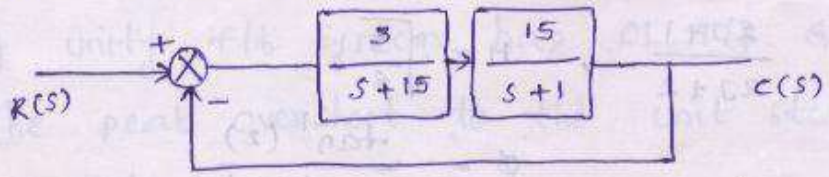
$$\Rightarrow s^2 + s(2 + kA) + k = 0$$

$\omega_n^2 = k = 16$

$2\xi\omega_n = 2 + kA$

$\Rightarrow A = 0.225$

Q. A block diagram shown in fig. gives a unity f/b CL control system. The ss error to the unit step i/p is −?



$$GH = \frac{45}{(s+1)(s+15)}$$

$$\frac{A}{1+k} = \frac{1}{1 + \frac{45}{15}} \times 100$$

$= 25\%$

Q. A control system is defined by the following mathematical exp. $\frac{d^2x}{dt^2} + 6 \cdot \frac{dx}{dt} + 5x = 12(1 - e^{-2t})$ The responce of the system at $t \to \infty$ - ?

$$X(s)[s^2 + 6s + 5] = 12\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

final value theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s \cdot F(s)$$

Initial value th

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s F(s)$$

Q. If the CL T/F of a control system is given by $\frac{C(s)}{R(s)} = \frac{1}{s+1}$ for the i/p $R(t) = \sin t$, the ss value of $c(t) = ?$

Sol:- (or) finding o/p or find response

linear system

$$r(t) = A\sin(\omega t \pm \theta)$$

$$= A\cos(\omega t \pm \theta)$$

$$c(t) = A \times M \sin(\omega t \pm \theta \mp \phi)$$

$$= \frac{1}{\sqrt{2}} \sin(t - \pi/4)$$

$\omega = 1$, $s = j\omega$

$$\frac{C(s)}{R(s)} = \frac{1}{j+1}$$

$M = 1/\sqrt{2}$ $\quad \angle\phi = \frac{1}{45^\circ}$

$\angle\phi = -\pi/4$

Q. for any linear system if i/p is a sinusoidal, the o/p also a sinusoidal but diff. in magnitude and phase angle. The standard form of i/p can be represented as

Sol: $\frac{C(s)}{R(s)} = \frac{s+1}{s+2}$, $r(t) = 10\cos(2t + 45^\circ)$

$$\Rightarrow \frac{2j+1}{2j+2}, \quad M = \sqrt{\frac{5}{8}}$$

$$\phi = \frac{\tan^{-1}(2)}{\tan^{-1}(2/2)}$$

$$= $$

$$c(t) = 10 \times \sqrt{\frac{5}{8}} \cos(2t + 63.45^\circ)$$

Q. Consider the unit step response of a unity f/b control system of OL T/f is $G(s) = \frac{1}{s(s+1)}$. The max. overshoot = ?

Sol: char. eq = $s^2+s+1=0$

$\omega_n = 1, \quad \xi = 0.5$

$\therefore \% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$

$= 0.163.$

Q. The CL T/f of a control system $\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+1)(s+2)}$ for a unit step i/p, the o/p is — ?

(1) 0 (2) ∞ ✓(3) $-3e^{-2t}+4e^{-t}-1$ (4) $-3e^{-2t}-4e^{-t}+1$

Sol. $C(s) = \frac{2(s-1)}{s(s+1)(s+2)}$

$\rightarrow \cancel{c(t)} = \frac{-1}{s} + \frac{4}{s+1} + \frac{-3}{s+2} = -1+4e^{-t}+3e^{-2t}$

Q. The L.T. of function $f(t) = F(s)$, $F(s) = \frac{\omega}{s^2+\omega^2}$

The final value of $f(t)$ = — ? $\quad f(t) = \sin\omega t$

(1) ∞ (2) 0 (3) 1 ✓(4) None

For sinusoidal signal the final value is None (does not have final value)

Q. A unity f/b system has OL T/f G(s), the SS error is zero for (a) step i/p type-1

(b). Ramp i/p type-1 (c) step i/p type-0

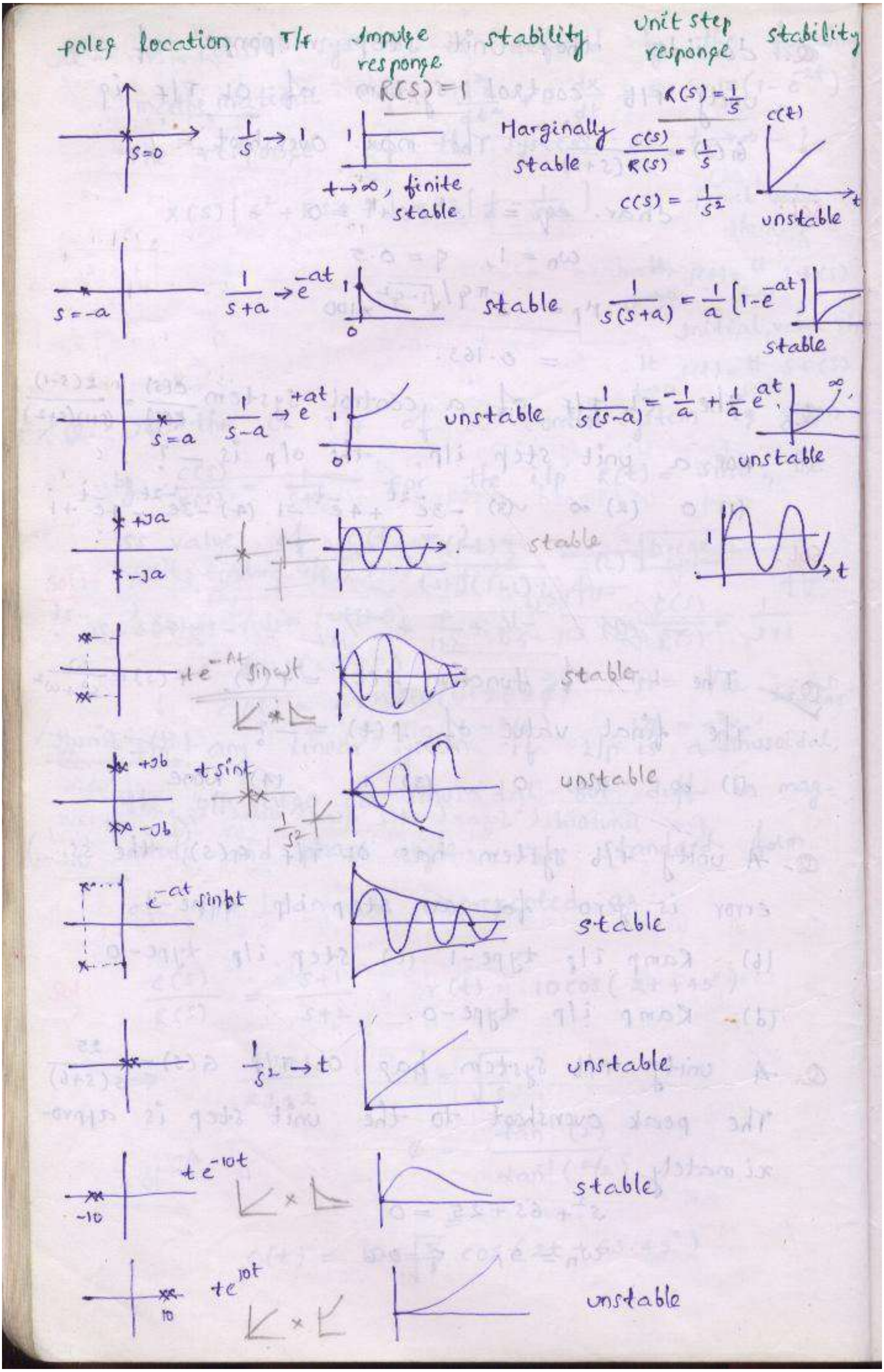
(d). Ramp i/p type-0.

Q. A unity f/b system has OL T/f $G(s) = \frac{25}{s(s+6)}$

The peak overshoot to the unit step is approximately. $\quad$ 10 %

$s^2+6s+25=0$

$\omega_n = 5, \quad \xi = 0.6$


poles location
T/f
Impulse response
stability
Unit step response
stability
$R(s) = 1$
$R(s) = \frac{1}{s}$
$s=0$
$\frac{1}{s} \to 1$
$t \to \infty$, finite
stable
Marginally stable
$\frac{C(s)}{R(s)} = \frac{1}{s}$
$C(s) = \frac{1}{s^2}$
$c(t)$
unstable
$s=-a$
$\frac{1}{s+a} \to e^{-at}$
stable
$\frac{1}{s(s+a)} = \frac{1}{a}\left[1-e^{-at}\right]$
stable
$s=a$
$\frac{1}{s-a} \to e^{+at}$
unstable
$\frac{1}{s(s-a)} = -\frac{1}{a} + \frac{1}{a}e^{at}$
unstable
$+ja$
$-ja$
stable
$te^{-At}\sin\omega t$
stable
$+jb$
$-jb$
$t\sin t$
$\frac{1}{s^2}$
unstable
$e^{-at}\sin bt$
stable
$\frac{1}{s^2} \to t$
unstable
te^{-10t}
-10
stable
te^{10t}
10
unstable

(* For unstable $e_{ss} = \infty$)

$\frac{t^2}{2} \cdot e^{10t} \sin 5t$

unstable

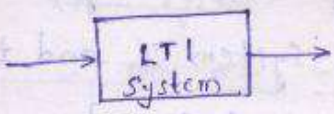
As type increases system stability decreases and e_{ss} improves.

RH — 4
RL — 2
BP — 3
NP — 1

06-06-07

1. CL stability
2. no. of CL poles
3. k
4. $k_{marginal}$, ω_{max}
5. Relation, stability T, ts
6. TF → RL Rules
7. CE NE → Gain ...

Stability :-



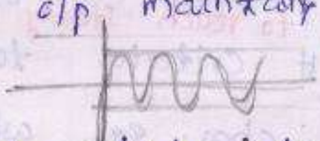
* A linear Time Invarient System is said to be stable, if the following conditions are satisfied.

(1). If the i/p is bounded to the system, the o/p must be bounded.

(2). If the i/p to the system is zero, the o/p must be zero, irrespective of all the initial condi.s.

Marginal / critical / Limitedly stable System:-

A LTI system said to be marginal, if for the bounded i/p the o/p maintains const. freq and amplitude.



The stability is classified into 2 ways.

1. Absolutely stable System
2. Conditional " "

Absolutely s. system :-

Here the system is stable for all the values of system parameters ie from k, 0 to ∞.

Conditional s. system :-

Here the system is stable for certain range of system parameters. ie $k > 0$, $k < 10, 20$..

Relative stability :-

R.S. depends on the position of closed loop poles. If the closed loop poles moves towards LHS, the R.S. improves. The R.S is applicable for only closed loop stable systems.

* The R.S. is used to find system τ and t_s. [how fast the transients are died out]

R.H. criteria:-

1. To find closed loop system stability.
2. To find no. of CL poles in the right half of s-plane.
3. To find range of k. value to find system stability.
4. To find $K_{marginal}$ value.
5. If the system is marginal stable to find the frequency of the oscillations. ($\omega_{marginal}$)
6. To find the relative stability ie τ & t_s & time required to reach steady state.

* In R.H criteria to find a CL system stability we use char. eq. where as in Root locus, BP, and NP uses OL T/F.

The n-order general form of char. eq. is

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots a_{n-1} s^1 + a_n s^0 = 0$$

$$\begin{array}{c|ccc}
s^n & a_0 & a_2 & a_4 \cdots \\
s^{n-1} & a_1 & a_3 & a_5 \cdots \\
s^{n-2} & \frac{a_1 a_2 - a_0 a_3}{a_1} & \frac{a_1 a_4 - a_0 a_5}{a_1} & \\
\vdots & \vdots & & \\
s^0 & a_n & &
\end{array}$$

1. To become a system stable all the coe. in the first column must be +ve. and no coe. should be zero.

2. If sign changes occurs in 1st column then the system is unstable. The no. of sign changes = no. of CL poles in the right half of the s-plane.

Q. Identify the system stability, for (1). $s^2+5s+10=0$ — stable

(2). $s^3+10s^2+3s+30=0$ — m.s. (3). $s^3+4s^2+5s+5=0$ — stable

(4). $s^3+8s^2+4s+100=0$ — unstable (5). $s^3+5s^2+10=0$ — missing 's' → unstable

* for $s^2+bs+c=0$, $b,c>0 \rightarrow$ stable

$b=0 \rightarrow$ m.s. (Marginal)

* for $as^3+bs^2+cs+d=0$, $ad=bc \rightarrow$ M.S

$bc>ad \rightarrow$ stable

missing term $\rightarrow$ unstable

$bc<ad \rightarrow$ unstable

Q. find the no. of poles in the right half of s-plane, for (i). $s^4+2s^3+6s^2+8s+10=0$

s^4	1	6	10
s^3	2	8	
s^2	2	10	
s^1	-2		
s^0	10		

2 sign changes so 2 poles on right half of s-plane

(ii). $s^4+2s^3+3s^2+4s+5=0$

s^4	1	
s^3		
s^2		
s^1		
s^0		

(iii). $s^4+2s^3+2s^2+4s+8=0$

s^4	1	2	8
s^3	2	4	
s^2	$\cancel{0}\ \epsilon$	8	
s^1	$\frac{4\epsilon-16}{\epsilon}$		
s^0	8		

If any zero occurs in the first column, replace zero by smallest +ve const and conti. Routh tabular form. Finally substitute $\epsilon = 0$ and check the no. of sign changes.

(iv). $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

(v). $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$

s^5	1	3	2
s^4	$1s^4$	$3s^2$	$2s^0$
s^3	$\cancel{0}\ 4$	$\cancel{0}\ 6$	0
s^2	$3/2$	2	
s^1	$2/3$		
s^0	2		

→ * whenever the poles are located symmetrical about original then the row of zero's occur.

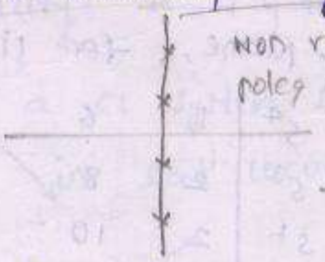
* whenever in Routh table, only once rows of zero are occured and all the coe. in 1st column +ve, then the CL poles must be on ima. axis which are symmetrical about origin.

* The auxillary eq. gives the location of the CL poles. The AE contains only even power of s- terms.

$AE \Rightarrow s^4 + 3s^2 + 2 = 0$

$\Rightarrow (s^2+2)(s^2+1) = 0$

$\Rightarrow s = \pm j\sqrt{2}, \pm j$. System is m.s.



Non repeated poles on ima axis

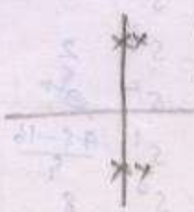
(vi). $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

s^6	1	4	5	2
s^5	3	6	3	
s^4	$2s^4$	$4s^2$	$2s^0$	
s^3	$\cancel{0}\ 8$	$\cancel{0}\ 8$	0	→ ①
s^2	$2s^2$	$2s^0$		
s^1	$\cancel{0}\ 4$	0		→ ②
s^0	2			

$2s^4 + 4s^2 + 2 = 0$

$s^4 + 2s^2 + 1 = 0$

$(s^2+1)^2 = 0 \Rightarrow s \pm j1$



Repeated poles on ima. axis

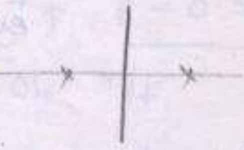
* whenever many times rows of zeros occurs and all the coe.s in the 1st column are +ve then the roots are repeated on ima. axis and which are symmetrical about origin and the system is unstable.

(vii). find the no. of CL poles in the left half of s-plane for $s^4 + s^3 - s - 1 = 0$.

s^4	1	0	-1
s^3	1	-1	
s^2	$1s^2$	-1	
s^1	$\cancel{0} \to 2 \longrightarrow ①$		
s^0	-1		

AE: $s^2 - 1 = 0$

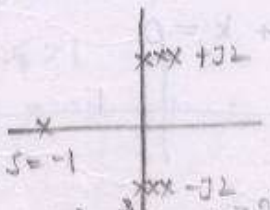
$s = \pm 1$



* whenever in the Routh table, row of zero's occurs and sign changes then the roots are located on the real axis which are symmetrical about origin.

(viii). find the Routh table for the given different poles location.

i).



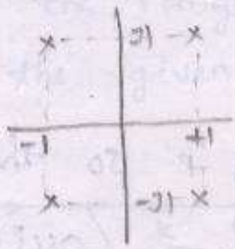
$(s^2+4)^2 (s+1) = 0$ need not to take

ii).

$(s^2+1)(s^2-1)(s^2+9) = 0$

iii).

iv).



$(s^2+2s+2)(s^2-2s+2) = 0$

Q. a). find the range of k value of system stability

b). find the k value to become the system m.s.

c). If the system is m.s. find the freq. of oscillations.

$s^3 + 9s^2 + 4s + k = 0 \Rightarrow 0 < k < 36$ (range)

$m = 36 \rightarrow$ m.s.

for freq. of oscillations,

even power of s terms = 0

$\Rightarrow 9s^2 + 36 = 0 \Rightarrow s = \pm j2$ rad/sec

(ii). $G(s).H(s) = \dfrac{k}{s(s+2)(s+4)(s+6)} + k$ char. eq

for $(s+1)(s+2)(s+3) = 0$ expansion

product of s terms	Addition of all const.	Σ of product of 2 const	Σ of π of 3 const.
s^3 +	$6s^2$ +	$11s$ +	6

char. eq $\Rightarrow 1 + GH = 0$

$\Rightarrow s(s+2)(s+4)(s+6) + k = 0$

$\Rightarrow s^4 + 12s^3 + 44s^2 + 48s + k = 0$

s^4	1	44	k
s^3	12	48	
s^2	40	k	
s^1	$\frac{40 \times 48 - 12k}{40}$		
s^0	k		

Q. Determine the value of k and P so that the system T/f $G(s) = \dfrac{k(s+1)}{s^3 + Ps^2 + 3s + 1}$ oscillates at a freq. of 2 rad/sec.

Sol. If freq. of oscillations are given so the system is m.s.

char. eq. $\Rightarrow s^3 + ps^2 + 3s + 1 + k(s+1) = 0$

$\Rightarrow s^3 + ps^2 + s(3+k) + 1 + k = 0$

$s^3 \quad 1 \quad 3+k$

$s^2 \quad p \quad k+1 \leftarrow$ AE

$s^1 \quad \frac{p(3+k)-(k+1)}{p} \rightarrow 0$ (M.S) $\rightarrow = 0 \Rightarrow p = \frac{k+1}{k+3}$

$s^0 \quad k+1$

AE : $ps^2 + (k+1) = 0$

$s = j\omega = j2$; $\Rightarrow p = \frac{k+1}{4}$ $\quad k = 1$

$\Rightarrow s^2 = -4$; $\Rightarrow -4p + (k+1) = 0$ $\quad p = 0.5$

Q. A unity f/b control system has an OL T/F

$G(s) = \frac{k(s+13)}{s(s+3)(s+7)}$. find the value of k for system stability (b). Determine the value of ξ, >, < or = 1 when k = 1.

Sol. $s^3 + 10s^2 + (21+k)s + 13k = 0$

$210 + 10k > 13k$

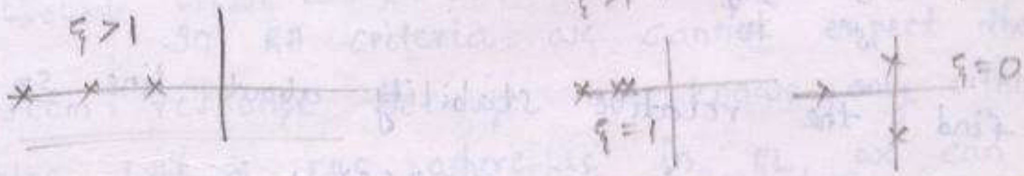
$210 > 3k$

$\Rightarrow 0 < k < 70$.

when k = 1,

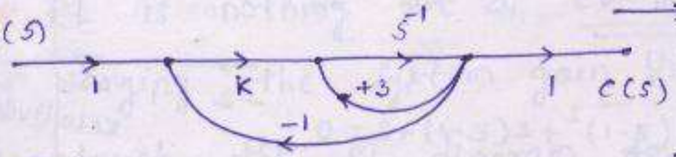
$s^3 + 10s^2 + 22s + 13 = 0$

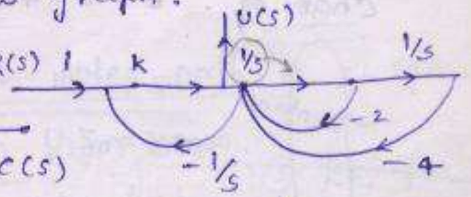
$\Rightarrow s = -1, -1.7, -7.2$



07-06-07

Q. find the range of k-value for system stability for the given signal flow graph.





$\frac{C(s)}{R(s)} = \frac{k/s}{1 - 3/s + k/s} = \frac{k}{s-3+k}$

$s^1 \quad 1$

$s^0 \quad k-3 > 0 \Rightarrow k > 3.$

T/F $= \frac{k}{1 + \frac{k}{s} + \frac{2}{s} + \frac{4}{s^2}} = \frac{ks}{s^2 + ks + 2s + 4}$

$s^2 \quad 1 \quad 4$

$s^1 \quad k+2$

$s^0 \quad 4 \rightarrow k > -2.$

Q. 

The f/b control system shown is stable.
(a) for all $k \geq 0$ (b). only if $k \geq 1$
(c) only if $0 \leq k \leq 1$ (d). only if $0 \leq k < 1$

$0 \leq k < 1$

char. eq

$1 + GH = 0$

$$\Rightarrow 1 + \frac{k(s-2)}{(s+2)^2} \cdot (s-2) = 0$$

$-1 < k < 1$

$$s^2(k+1) + s(4-4k) + 4k + 4 = 0$$

s^2	$k+1$	$4k+4$
s^1	$4-4k \rightarrow > 0 \Rightarrow k < 1$	
s^0	$4k+4 \rightarrow > 0 \Rightarrow k > -1$	

Q. The loop gain GH of a CL system is given by the following eq. $GH = \frac{k}{s(s+2)(s+4)}$ The value of k for which the system just unstable is -

(M.S)

$$s^3 + 6s^2 + 8s + k = 0$$

$$\Rightarrow k = 48 \longrightarrow \text{for M.S.}$$

Q. The char. eq. of a f/b control is $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$. The no. of roots in the right half of s-plane - ?

Ans: 2

s^4	2	3	10
s^3	1	5	
s^2	-7	10	
s^1	$\frac{45}{7}$		
s^0	10		

Q. find the relative stability about line $s = -1$ for

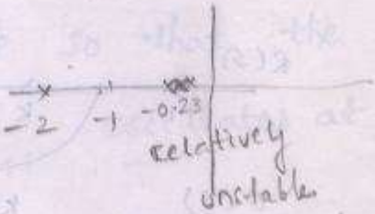
$$G(s) = \frac{2}{s(s+1)(s+2)} \quad \& \quad H(s) = 1$$

R.S. only applicable to stable system

$$\text{char. eq.} = s^3 + 3s^2 + 2s + 2 = 0 \longrightarrow \text{stable}$$

$z = s+1$

$\rightarrow s = z - 1$



$$(z-1)^3 + 3(z-1)^2 + 2(z-1) + 2 = 0$$

$$\Rightarrow z^3 - z + 2 = 0$$

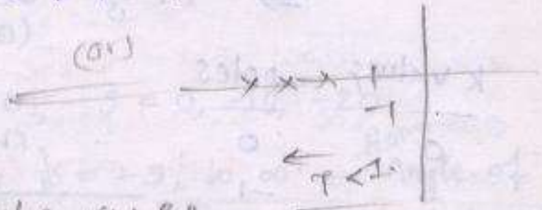
z^3	1	-1
z^2	0ϵ	2
z^1		
z^0		

Relatively unstable

Q. check whether the T is greater or lesser or equal to 1 sec. for $s^3 + 7s^2 + 25s + 39 = 0$.

$s = -\frac{1}{T}$ (or)

$s = -1$



$s = z - 1$ sub. and then solve using R.H.

Q. If the RH criteria applicable, is applicable for
* sine & cosine terms - ?

The RH criteria not applicable for trigonometric terms and exponential terms b'coz gives infinite series. but approximate soln. can be obtained for exponential terms. transpotation system

Q. find the system stability for $G(s) = \frac{e^{-sT}}{s(s+1)}$, $H(s) = 1$

Transpotation delay system not effect the magnitude it effects

$$e^{-sT} \approx (1 - sT)$$

$$G(s) = \frac{(1 - sT)}{s(s+1)}$$

char. eq $= s^2 + s + 1 - sT = 0$

s^2	1	1
s^1	$1-T$	$\rightarrow > 0$
s^0	1	$\rightarrow T < 1\,sec$

Root Locus :-

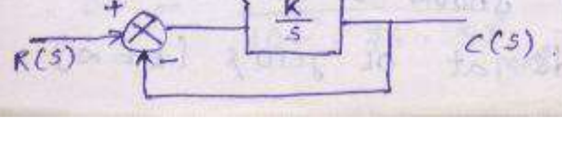
↳ CL pole ↳ path → k = 0 to ∞

In RH criteria we cannot expect the system response because we knows only either poles LHS or RHS where as in RL, we can find the system response by observing the CL poles path.

* RL is nothing but a CL poles path by varying the system gain from 0 to ∞.

RL
1. CL sys
2. ∠k∠
3. kmar / ωmar
4. k & ondamped / over / critical
5. RS
6. ... = 1 inclination ... = cos

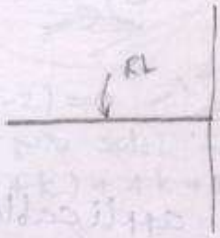
Q. construct the RL diagram for the following block diagrams.



Char. eq $\Rightarrow 1 + GH = 0$

$\Rightarrow 1 + \frac{k}{s} = 0 \rightarrow s + k = 0 \Rightarrow$ CL poles $s = -k$

k values	poles
0	0
1	-1
10	-10
⋮	
∞	$-\infty$



for $G = k/s^3$

$s^3 + k = 0$

$s = \sqrt[3]{k}$

$= -1\sqrt[3]{k}$

for $G = k/s^2$

$1 + k/s^2 = 0$

$s^2 + k = 0$

$s = \pm j\sqrt{k}$

k	s
0	0
1	$\pm j1$
10	$\pm j10$

* As order increases drawing the RL diagram with char. eq. becomes very difficult hence OL T/f is used to draw a RL.

⇒ Relationship b/w OL T/f poles and zero's to CL T/f poles.

OL T/f $G(s) \cdot H(s) = \frac{k \cdot N(s)}{D(s)}$ — ①

OL zero's $N(s) = 0$

OL poles $D(s) = 0$

CL poles $1 + G(s) \cdot H(s) = 0$

$1 + k \cdot \frac{N(s)}{D(s)}$ → (0 to ∞)

$\Rightarrow D(s) + k\, N(s) = 0$

* CL poles are nothing but a sum of OL poles and OL zero's with system gain k.

* Case 1: $k = 0$

$\Rightarrow D(s) = 0 \rightarrow$ CL poles

when $k = 0$, the OL poles must be equal to CL poles.

* Case 2: $k = \infty$, $N(s)$ must be zero. $N(s) = 0$

so OL zero's = CL poles $k = -\frac{D(s)}{N(s)}$

* The RL diagram should be start at $k = 0$ [at OL poles] and ends at OL zero's $[k = \infty]$

Q. find where the RL diagram starts and ends.

$$G(s) \cdot H(s) = \frac{k(s+5)}{s(s+10)(s+20)}$$

starts: OL poles $k=0$, $s = 0, -10, -20$

Ends: OL zero's $k=\infty$, $s = -5, \infty, \infty$ ← Along Angle of Asymptote dirc.

⇒ Angle & Magnitude Condition :-

* The CL system stability is given by char. eq. $1 + GH = 0$.

The construction rules of RL are obtained from angle & magnitude condition.

$\Rightarrow G(s) \cdot H(s) = -1 + j0$.

But the RL diagram drawn for O/L T/f ie $GH = -1+j0$ $-1+j0$

Angle condition: $\angle G(s) \cdot H(s) = \angle -1 + j0$ (± 360) $2(360)$ $4(360)$

$= \pm(2q+1)180^\circ, q = 0, 1, 2 \ldots$

purpose :-

$=$ odd multiples(± 180)

To check any point existing on RL or not that means all the points on RL must satisfy the angle condition.

Q. Check whether the following points lies on root locus or not for $GH = \frac{k}{s(s+2)(s+4)}$

① $s = -0.75$ ② $s = -1 + j4$

$$\angle GH = \frac{\angle k}{\angle s \; \angle s+2 \; \angle s+4}\Bigg|_{s=-0.75}$$

$$= \frac{\angle k}{\angle -0.75 \; \angle 1.25 \; \angle 3.24} = \frac{0^\circ}{\pm 180^\circ \cdot 0^\circ \cdot 0^\circ}$$

$\tan 0 = 0$

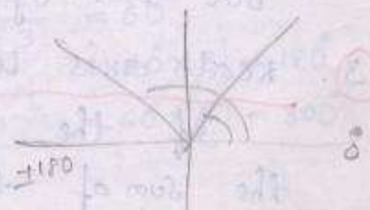
$\tan 180 = 0$

b'coz of -ve sign

$= \pm 180^\circ$ satisfies angle condi. so the given point on RL.

for $s = -1 + j4$

$$\angle GH = \frac{\angle k}{\angle(-1+j4) \; \angle(1+j4) \; \angle(3+j4)}$$



$= \frac{0}{104^\circ \cdot 76^\circ \cdot 53^\circ}$ not satisfies, so the given point not on RL.

→ Magnitude condition :- $G(s).H(s) = -1+j0$ which is $|G(s).H(s)| = 1$.

The magnitude of GH at a point on the Root locus which is ~~that~~ that means the magnitude condi. is valid only when the given point is on the RL.

purpose:- To apply mag. condi. 1st we've to verify angle condi.

To find the system gain at any point which is on the RL.

Q. Consider the system with $GH = \frac{k}{s(s+4)}$. Find K of system gain at a point $s = -2+j5$.

Sol: Angle condi. $\angle GH = \frac{\angle k}{\angle(-2+j5)\angle(+2+j5)} = -180°$

satisfies angle condi. so the given point is on RL.

To find k, magnitude condi.

$$M.C. \quad \frac{k}{\sqrt{4+25}\sqrt{4+25}} = 1 \Rightarrow k = 29.$$

Rules for constructing RL:-

① Symmetrical :-

The RL diagrams are symmetrical about real axis because the loc. of poles and zero's are symmetrical about real axis.

②. No. of RL branches / Loci :-

proper T/F → If the poles $P > Z \Rightarrow$ no. of RL branches $(N) = P$

Improper T/F → $P < Z \Rightarrow$ " $= Z$

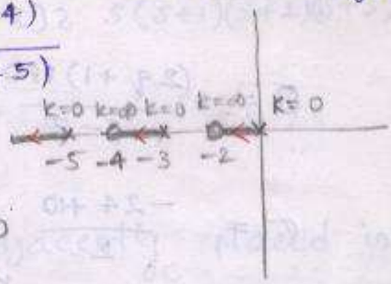
But actually $N = P = Z$. ← strictly proper T/F.

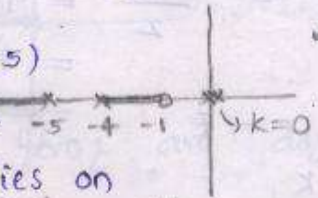
③. Real axis loci :-

If the point exists on real axis RL branch the sum of the poles and zero's to the RHS of that point should be odd.

Q. Find the sections of real axis which belongs to RL. (1). $GH = \frac{K(s+2)(s+4)}{s(s+3)(s+5)}$

(2). $GH = \frac{K(s+1)}{s^2(s+4)(s+5)}$





Check whether the following points lies on RL or not - (a). 0 (b). -1, (c). -4 (d). -5 (e). -2, (f). -∞

no, yes, yes, yes, yes, yes, yes

* At the initial position of p & z's there must be a RL branch.

④ Asymptode Angles :-

Asymptotes are RL branches which approach to ∞.

* The no. of asymptotes = p - z.

* Angle of asymptote = $\frac{(2q+1)180}{p-z}$, $q = 0, 1, \ldots (p-z-1)$.

→ The angle of asymptote gives the direction of the zeros when the p > z.

→ The asymptotes are symmetrical about real axis.

⑤ Centroid :-

Centroid gives the intersection point of asymptotes on the real axis.

$$\sigma = \frac{\text{sum of real part of poles} - \Sigma R.P(z's)}{P-Z}$$

The 'σ' may be located any where on real axis.

Q. Calc. the angle of asymptotes and σ for

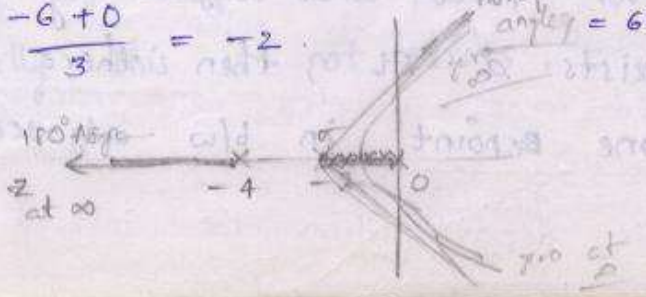
$$GH = \frac{k}{s(s+2)(s+4)}$$

$$\theta = \frac{(2q+1)180}{p-z} \rightarrow \frac{180}{p-z} = \frac{180}{\boxed{3}} = 60°$$

$$= 60° \times \boxed{3} = 180°$$

$$\text{no. of angles} = 60° \times \boxed{5} = 300°$$

$$\sigma = \frac{-6+0}{3} = -2.$$

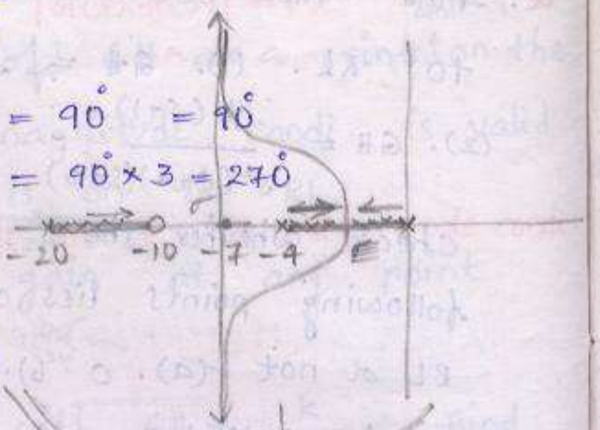


(2). $GH = \frac{K(s+10)}{s(s+4)(s+20)}$

$\theta = \frac{(2q+1)180}{P-Z} = \frac{180}{2} = 90^\circ = 90^\circ$

$= (90^\circ \times 3 = 270^\circ$

$\sigma = \frac{-24+10}{2} = -7$



(3). $GH = \frac{K}{s(s+1)(s+2)(s+3)}$

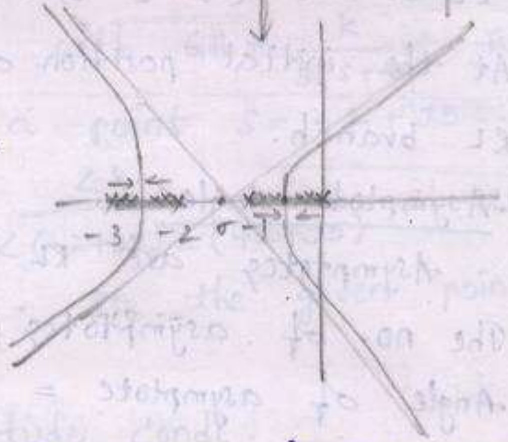
$\theta = \frac{180}{4} = 45^\circ$

$45 \times 3 = 135^\circ$

$45 \times 5 = 225^\circ$

$45 \times 7 = 315^\circ$

$\sigma = \frac{-6}{4} = -1.5$



⑥. Break points :-

A point where the RL meets, intersection point of RL branches. It is point where RL branches leaves or enter into the real axis.

→ The point where RL branches leaves the real axis - break out point

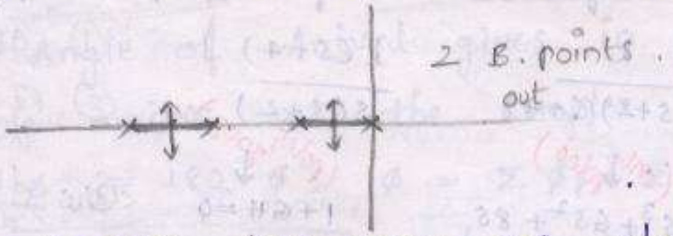
→ The point where RL branches enter into the real axis - break in point.

* The RL branches enter or leaves real axis with an angle of $\pm \frac{180^\circ}{n}$ where 'n' is no. of RL branches [no. of poles at the break point].

→ pointing the existance of B.Points :-

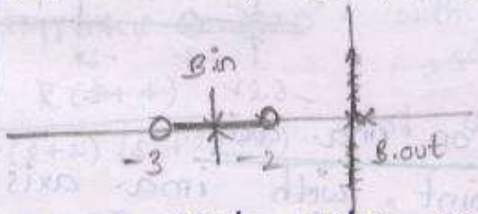
(1). whenever poles are adjasently placed in b/w there exists a RL, then there should the min. one Br.point in b/w adjascently placed poles.

Q. find the B. points for $GH = \frac{K}{s(s+1)(s+2)(s+3)}$



(2) whenever two zeros are adjacently placed in b/w there exists the RL branch then there should be the min. one B.in point in b/w adj. placed zero's.

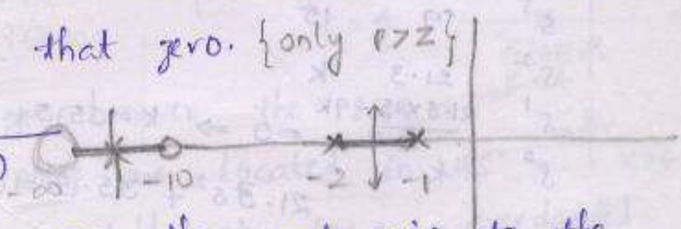
Q. find the no. of B. points for $GH = \frac{K(s+2)(s+3)}{s^2}$



whenever multiple poles or zeros located at a particular loc. then there must be the atleast one break away or break in point at that loc.

(3) whenever zero exists left most side on real axis, to the left of that zero there exists a root locus branch then there should be the min. one B.in point to the left of that zero. {only $P > Z$}

$GH = \frac{K(s+10)}{(s+1)(s+2)}$



(4) when pole lies on the real axis to the left of that pole there exists a RL branch there should be the min one B.away point to the left of that pole when $P < Z$ only. practically this is not exists.

Q. Determination of co-or. of B. points :-

GH $\frac{k}{s(s+2)}$, $\frac{k}{s(s+2)(s+4)}$, $\frac{k(s+4)}{s(s+2)}$.

(only poles) (only poles) (poles & zeros)

① CE

② Rewrite CE in the form k = f(s)

③ $\frac{dk}{ds}=0$

④ Roots of $\frac{dk}{ds}$ gives valid & invalid B. point. valid B. point should be on RL for which k → +ve

diffe. $s^2+2s=0 \Rightarrow 2s+2=0 \Rightarrow s=-1$

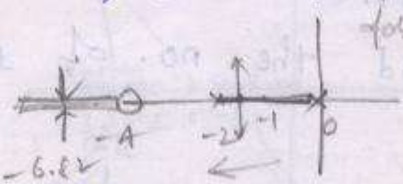
$s^3+6s^2+8s \xRightarrow{d/ds} 3s^2+12s+8=0 \Rightarrow s=-0.84, -3.15$

$1+GH=0 \Rightarrow GH=-1$

$\therefore -1=\frac{k(s+4)}{s(s+2)}$

$\frac{dk}{ds}=\frac{(-2s-2)(s+4)+s^2+2s}{(s^2+2s)(s+4)}=0$

$\Rightarrow s=-1.17, -6.82$



⑦. Intersection point on ima. axis :-

Intersection point with ima. axis given by RH criteria.

when $k_{marginal} \rightarrow +ve$, there will be I. points.

① CE
② RH Criteria
③ k_{mar}
④ from AE
⑤ Roots AE

Eg :- $GH=\frac{k}{s(s+1)(s+3)(s+5)}$

$\rightarrow CE = s^4+9s^3+23s^2+15s+k=0$

s^4	1	23	k
s^3	9	15	
s^2	21.3	k	
s^1	$\frac{21.3\times15-9k}{21.3}$		
s^0	k		

Make even powers to zero

$\frac{21.3\times15-9k}{21.3}=0 \Rightarrow k=35.5$

$21.3s^2+35.5=0$

$\Rightarrow s=\pm j\,1.29$

⑧. Angle of departure and arrival :-

Angle of departure should be calculated at complex conjugate poles and angle of arrival calculated at complex conjugate zero's.

* Angle of departure gives with what angle the pole depart from the initial position.

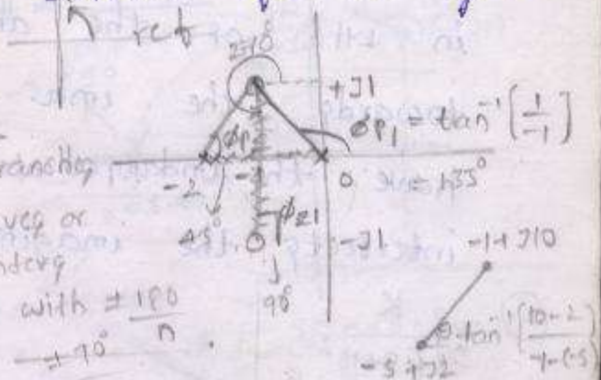
$\phi_d = 180 - \phi \; ; \; \phi = \Sigma \phi_p - \Sigma \phi_z$

* Angle of Arrival gives in what dirc. the pole arrives at the complex zero.

$\phi_a = 180 + \phi \; ; \; \phi = \Sigma \phi_p - \Sigma \phi_z$

Q. Find the angle of arrival for the system

$$GH = \frac{K(s^2+2s+2)}{s(s+2)}$$



$\phi = 135 + 45 - 90$

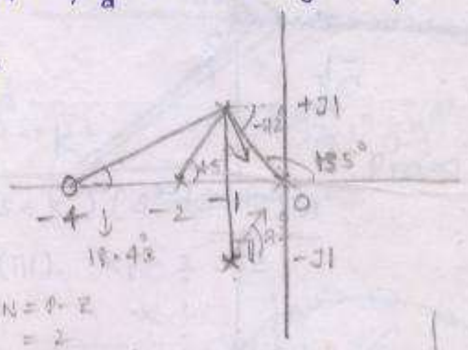
$= 90$

$\phi_a = 180 + \phi = 270^\circ$

RL branches leaves or enters with $\pm \frac{180}{n}$

$\pm 90^\circ$

Q. $GH = \frac{K(s+4)}{s(s+2)(s^2+2s+2)}$ find ϕ_d at conju. poles.



$\phi = 135 + 90 + 45 - 18.43^\circ$

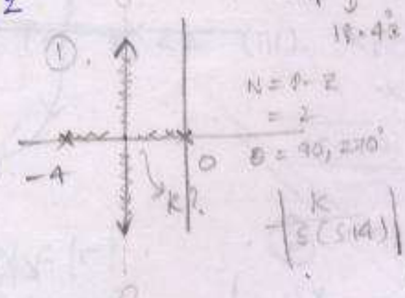
$= 251.5^\circ$

$\phi_d = 180 - 252$

$= -72^\circ$.

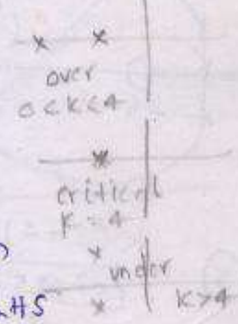
Find out Equivalent RL

1. $GH = \frac{K}{s(s+4)}$



$\left|\frac{K}{s(s+4)}\right| = 1$

$K = 4$



2. $\frac{K}{s(s+1)^2}$

3. $\frac{K(s+5)}{s^2}$

4. $\frac{K(s^2+2s+2)}{(s+4)(s+6)}$

5. $\frac{K(s+4)(s+6)}{s^2+2s+2}$

6. K/s, K/s^2, K/s^3, K/s^4

7. $\frac{K}{s(s+1)^2(s+2)}$

8. $\frac{K(s+1)^2}{s(s+2)}$

9. $\frac{Ks}{s^2+4}$

10. $\frac{K}{s(s^2+2s+2)}$

* whenever the system poles are located in LHS at different loc.s → overdamped.

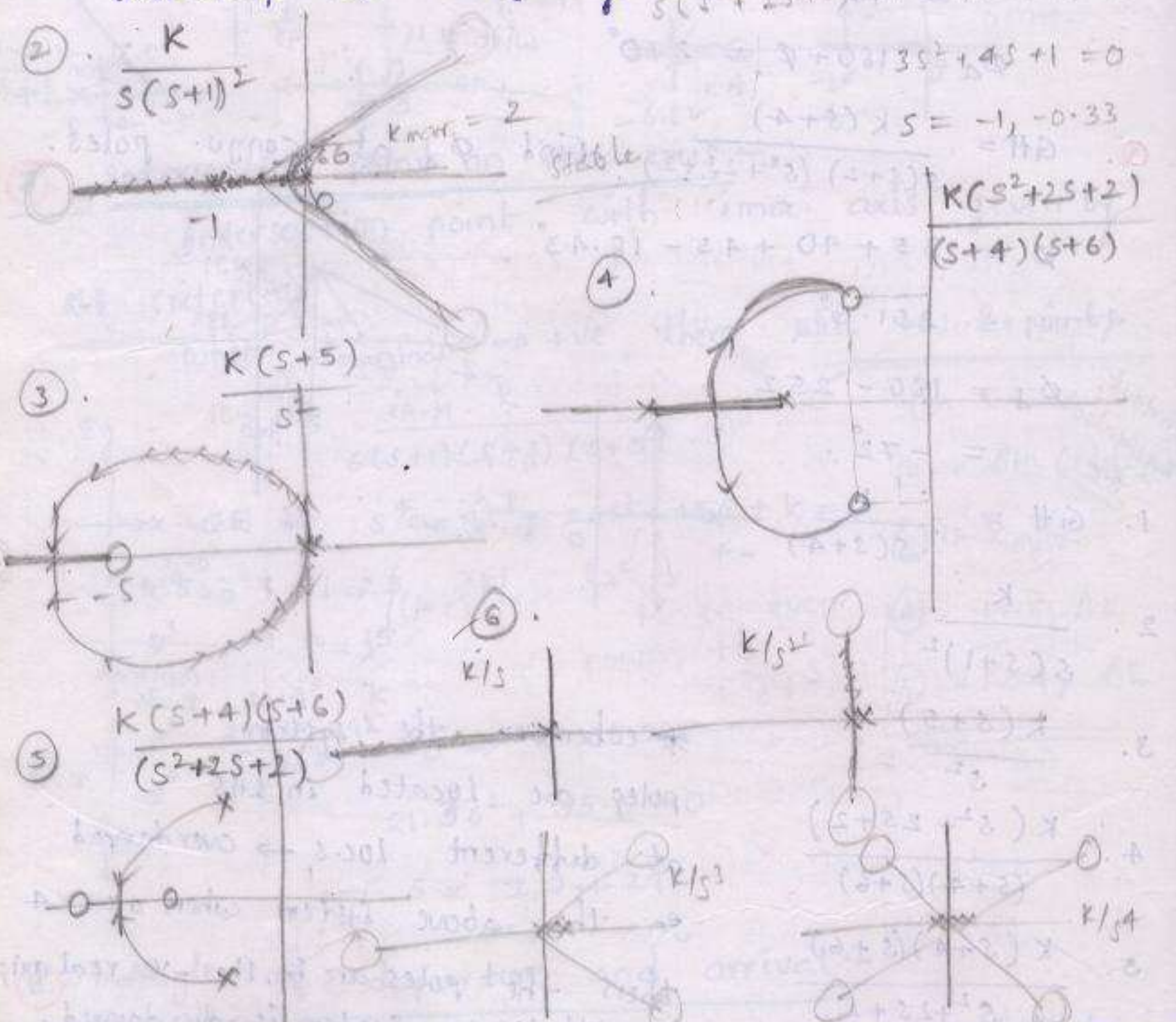
In the above system when $0 < K < 4$ then the poles are in the -ve real axis at diff loc.s, system is overdamped.

* whenever the system having B. point or roots meet at a particular point then the system is critical damped.

In the above system when $K = 4$ both poles met at $s = -2$.

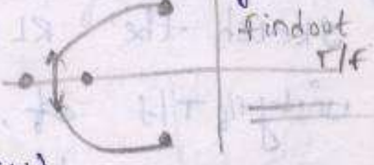
* whenever the RL branches leaves or enters into the real axis, the system should have the under damped nature.

* whenever the angle of asymptotes $< 90^\circ$ and σ in LHS or the angle of departure and arrival towards the ima axis then the system should have the undamped nature. The RL branches meets or intersects the imaginary axis.



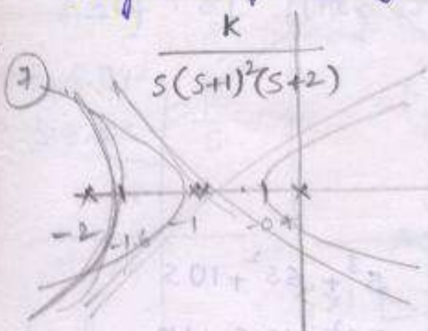
→ when given a RL diagram, to find T/F, observe the direction of RL branches. If the RL branch away from point then the point is pole. If the RL branch inside the point or towards the point then the point is zero.

* Whenever the T/F consists only poles at origin the RL diagrams are nothing but angle of asymptotes.



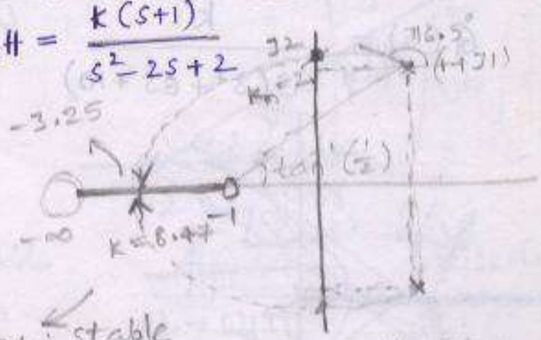
findout T/f

$\frac{K}{s(s+1)^2(s+2)}$



11).

$GH = \frac{K(s+1)}{s^2-2s+2}$



when $K>2$; stable

undamped $K=2$ m.s.

$\phi = 90 - 26.56$

$\phi_d = 116.5^\circ$

under damped $\leftarrow 2<K<8.47$

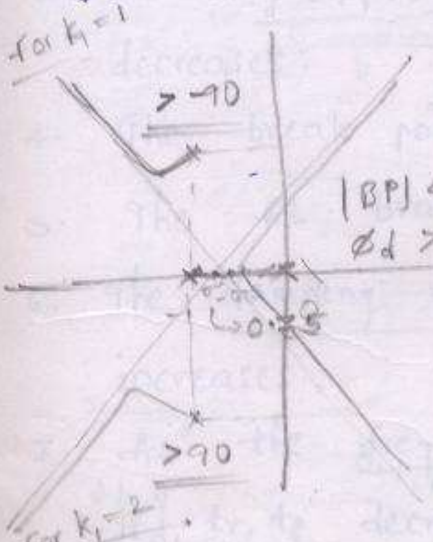
$K = 8.47 \rightarrow$ critical

$K > 8.47 \rightarrow$ over damped.

Refer once again

Q. The OL T/f $GH = \frac{K}{s(s+k_1)(s^2+2s+2)}$. Draw RL for (i) $k_1>2$ (ii). $k_1<2$ (iii). $k_1=2$.

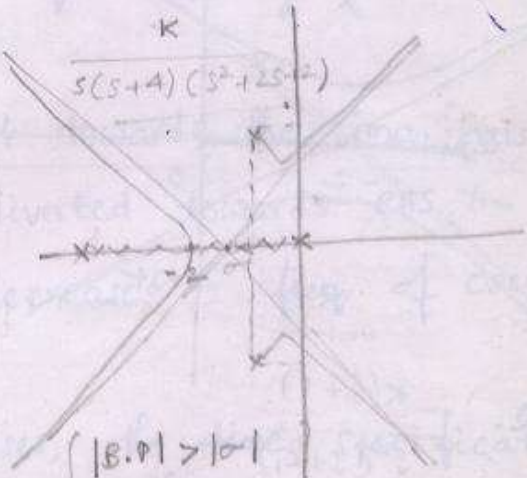
for $k_1=1$



$|BP| < |\sigma|$

$\phi_d > \mp 90^\circ$

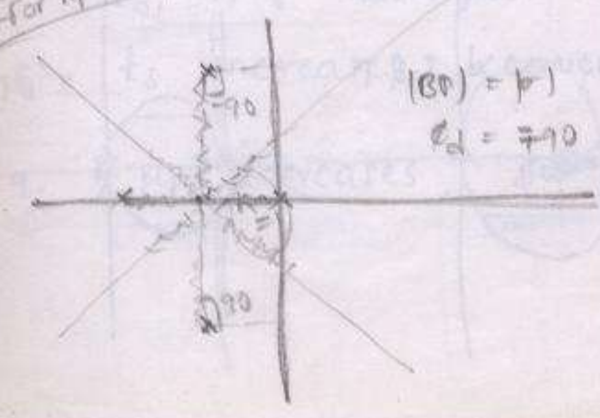
$\frac{K}{s(s+4)(s^2+2s+2)}$



$|B.P| > |\sigma|$

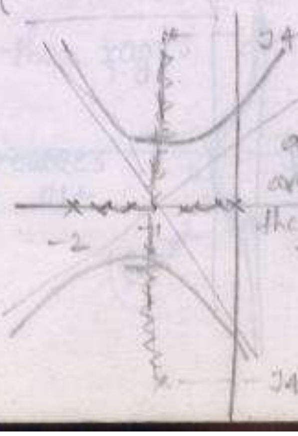
$\phi_d < \mp 90$

for $k_1=2$



$|BP| = |\sigma|$

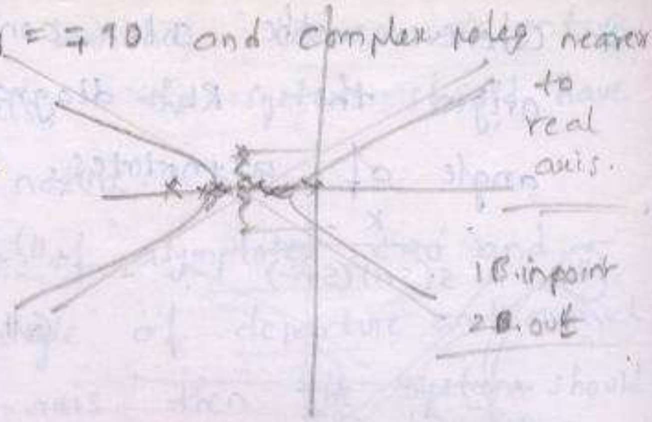
$\phi_d = \mp 90$



$|BP| = |\sigma|$

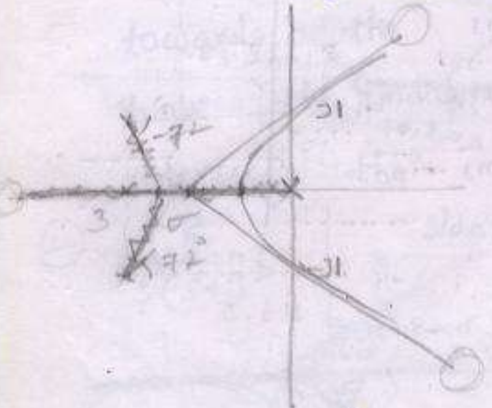
$\phi_d = \mp 90$

and complex poles are away from the real axis when $|\beta| = |\sigma|$, $\phi_d = \mp 90$ and complex poles nearer to real axis.



Q. Sketch the RL for unity f/b T/f of,

$$G(s) = \frac{k}{s(s^2+6s+10)}$$



$s^3 + 6s^2 + 10s$

$3s^2 + 12s + 10$

$-1.18, \ -2.81$

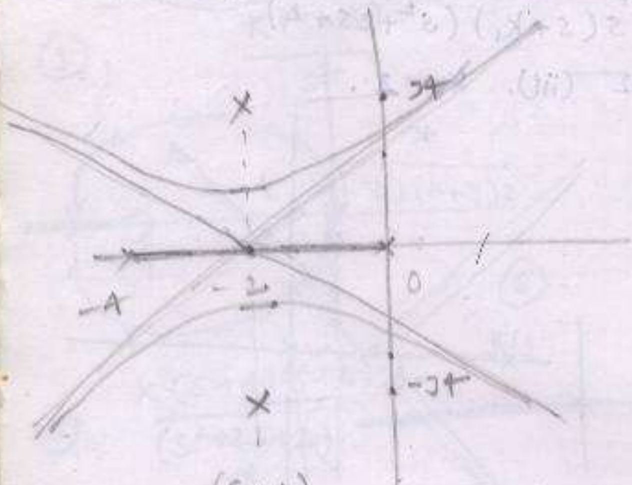
$\sigma = \frac{-6}{3}$

$= -2$

$\phi_p = 90 + \tan^{-1}(\frac{1}{...}$

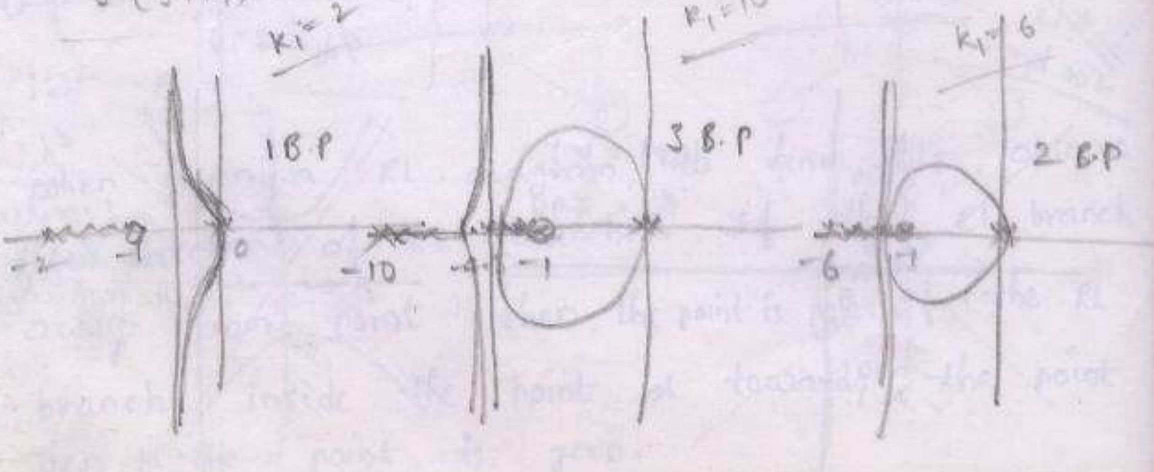
$\phi_d = -72°$

Q. $GH = \dfrac{k}{s(s+4)(s^2+4s+20)}$



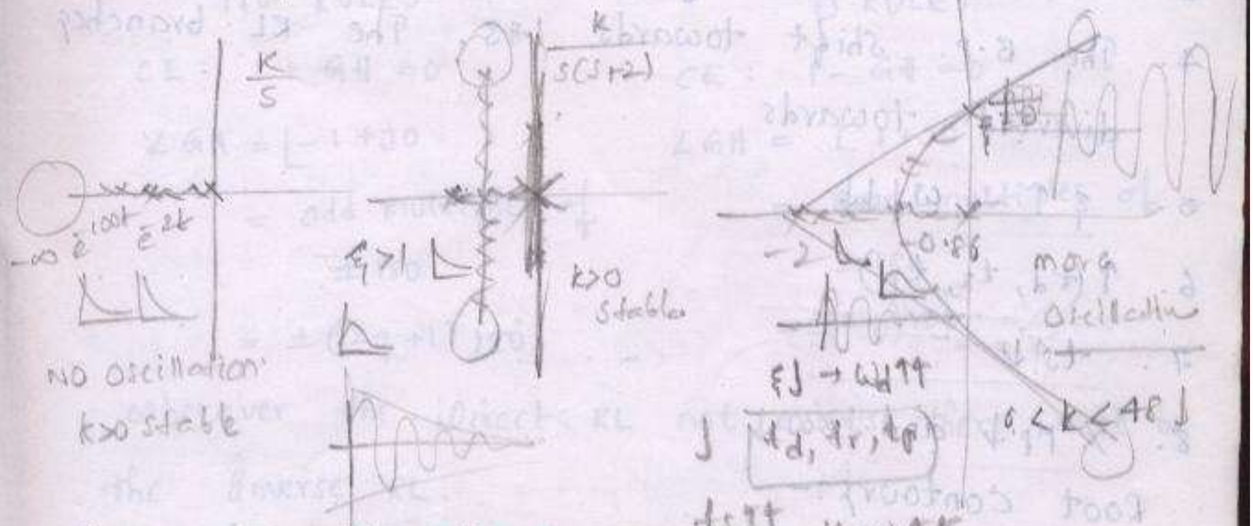
$-2 \pm j2.45$

Q. $\dfrac{K(s+1)}{s^2(s+k_1)}$



Effects of Addition of poles & zero's:-

The addition of P/Z's always in the left half of the s-plane.



(1). Addition of poles:-

1. The system becomes more oscillatory.
2. The system relative stability decreases.
3. The range of k-value for the system stability decreases.
4. The break point shift towards the ima. axis.
5. The RL branches diverted towards RHS.
6. The damping factor decreases, freq. of osci. increases.
7. As the freq. increases, the time specification t_d, t_r, t_p decreases.
8. t_s increases because the roots are moving
9. %M_p increases, BW increases. (BW ∝ 1/tr)

(ii). Addition of zero's :-

1. The system becomes less oscillatory.
2. The range of k value's for system stability increases
3. The relative stability increases.
4. The B.P. shift towards LHS. The RL branches diverted towards
5. $\xi \uparrow - \omega_d \downarrow\downarrow$
6. $\uparrow (t_d, t_r, t_p)$
7. $t_s \downarrow$
8. $\% M_p \downarrow$ and $BW \downarrow$

Root contours :-

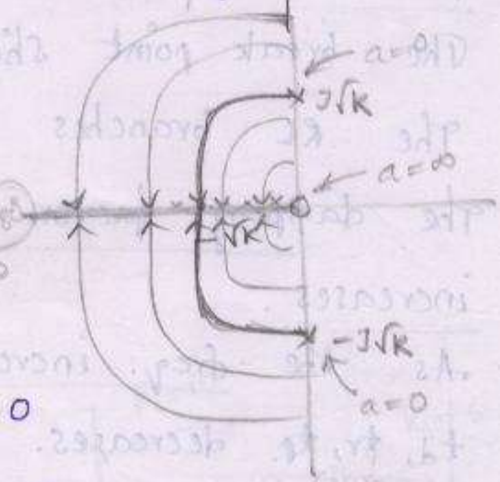
If the T/F or char. eq. contains more than one unknown parameter, varying all the parameters from 0 to ∞, and drawing a RL diagram is nothing but a RC.

Draw the RC for the following CE : $s^2+as+k=0$

Assume a : system gain

k : const.

$$GH = \frac{as}{s^2+k}$$



$$-1 = \frac{as}{s^2+k}$$

$$a = \frac{-s^2-k}{s} \qquad \frac{da}{ds} = 0$$

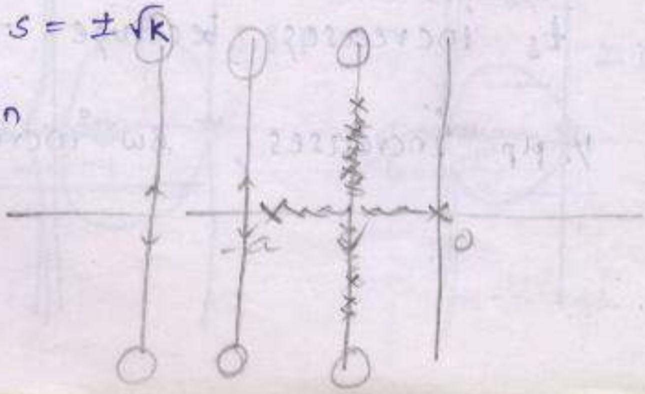
$$\Rightarrow s = \pm\sqrt{k}$$

Assume :

k : system gain

a : const.

$$GH = \frac{k \rightarrow 0 \text{ to } \infty}{s(s+a)}$$



Difference b/w direct RL and Inverse RL :-

	Direct RL	Inverse RL
1.	$k \rightarrow 0$ to ∞	$k \rightarrow -\infty$ to 0
	180° RULES	0° RULES
	CE : $1 + GH = 0$	CE : $1 - GH = 0$
	$\angle GH = \angle -1 + j0$	$\angle GH = \angle 1 + j0$
	= odd multiples of ± 180	= Even multiples of $\pm 180^\circ$
	$= \pm(2q+1)180^\circ$	$= \pm(2q)180^\circ$

wherever the Direct RL not exists there must be the Inverse RL.

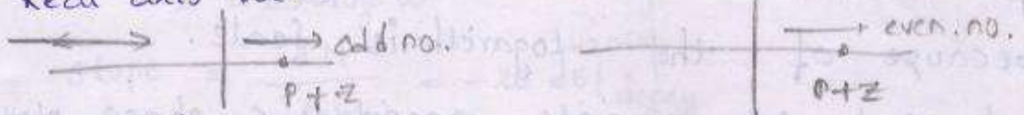
	Symmetry	Symmetry

2. no. of loci

$P > Z \Rightarrow N = P$

$P < Z \Rightarrow N = Z$

3. Real axis loci.

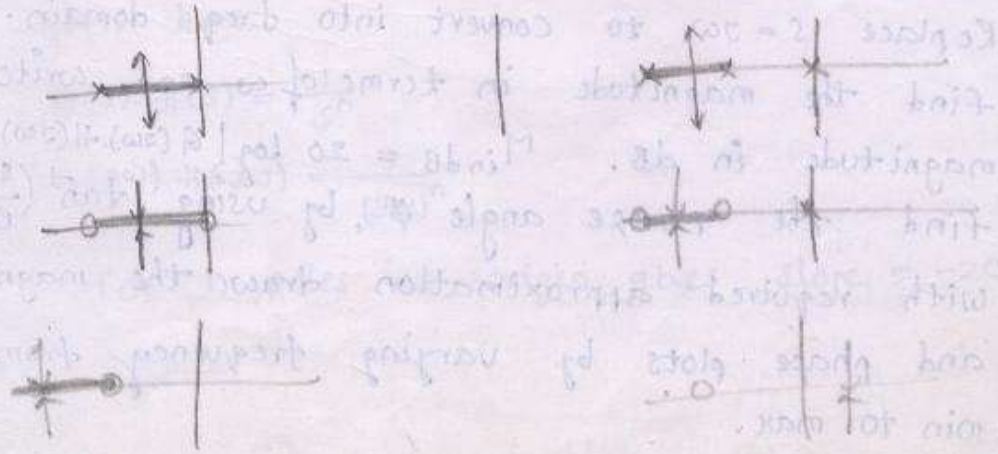


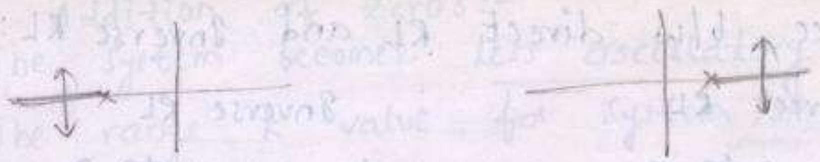
4. Asymptotes :-

$$\theta = \frac{(2q+1)180}{P-Z} \qquad\qquad \theta = \frac{(2q)180}{P-Z}$$

5. $$\sigma = \frac{\Sigma \text{ R.P. poles} - \Sigma \text{ R.P. zero's}}{P-Z}$$

B. Points





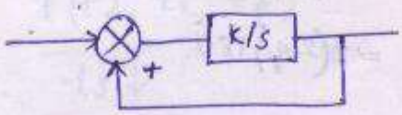
0 to ∞ → k   ∞ to 0 ← k

$\phi_d = 180 - \phi$ $\phi_d = 0 - \phi$

$\phi_a = 180 + \phi$ $\phi_a = 0 + \phi$

gr with

$K_{(max)}$ +ve $K_{(max)}$ −ve



$(1 - GH = 0)$

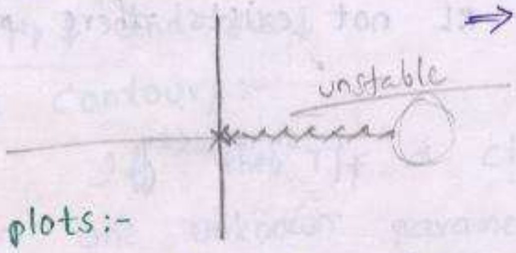
CE : $s - k = 0$

$\Rightarrow s = k.$

K/s^2

$s^2 - k = 0$

$s = \pm\sqrt{k}$



Bode plots:-

1. we can draw the bode plot for any higher order system and can be find the CL system stability. because of the logarithmic scale.
2. The bode plot consists magnitude & phase plots.

purpose:

1. freq. response OL T/f
2. CL system stability
3. Gm & Pm

Procedure to draw Bode plots:-

1. Replace $s = j\omega$ to convert into freq. domain.
2. find the magnitude in terms of ω and write magnitude in dB. $M_{in\,dB} = 20 \log |G(j\omega).H(j\omega)|$
3. find the phase angle ϕ, by using $\tan^{-1}\left(\frac{\text{Ima. Part}}{\text{Real part}}\right)$
4. with required approximation draw the magnitude and phase plots by varying frequency from min to max.

Q. $G(s)H(s) = k$

$G(j\omega)H(j\omega) = k$

$M = k$

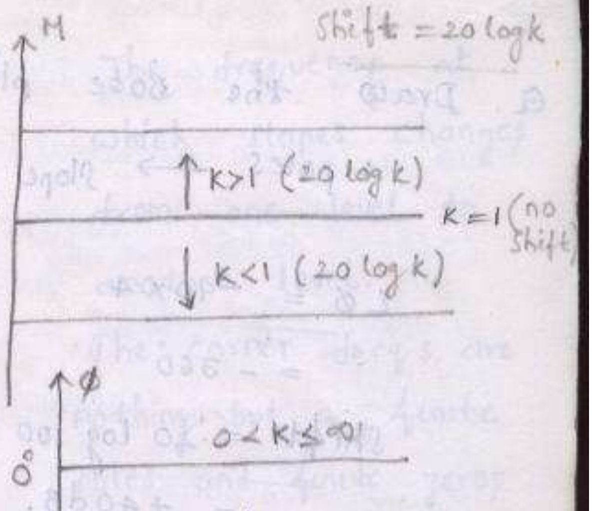
$M_{in\,dB} = 20 \log k \xrightarrow{} \text{slope} = 0$

$k = 1, \quad M = 0\text{ dB}$

$k = 10, \quad M = +20\text{ dB}$

$k = 0.1, \quad M = -20\text{ dB}$

$\angle G(j\omega).H(j\omega) = \angle k \longrightarrow$



* The phase plot is always ind. of k value, where as the shift in Magnitude plots depends on k - value.

Q. $G(s).H(s) = \frac{1}{s}$

$G(j\omega).H(j\omega) = \frac{1}{j\omega}$

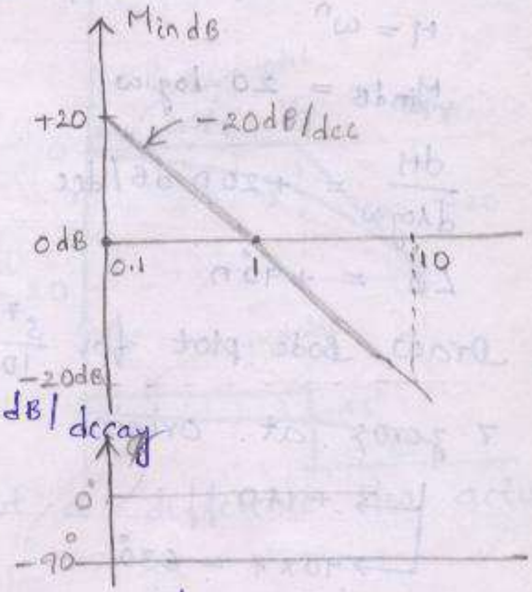
$M = 1/\omega$

$M_{in\,dB} = 20 \log \frac{1}{\omega}$

$= -20 \log \omega$

$\text{slope} = \frac{dM}{d\log\omega} = -20\text{ dB/decay}$

$\angle\phi = \frac{\angle 1}{\angle j\omega} = -90^\circ$



NOTE:- whenever the T/f consists of poles and zeros at the origin then the plot start at opposite sign of the slope and intersect 0dB line at $\omega = 1$, when $k = 1$.

$G(s).H(s) = \frac{1}{s^n}$

$G(j\omega)H(j\omega) = \frac{1}{(j\omega)^n}$

for n poles at origin gives slope $= -20 \times n$ dB/decay

Q. Draw the Bode plot for $\frac{100}{s^4}$.

4 poles $\longrightarrow$ slope $= -20 \times 4$

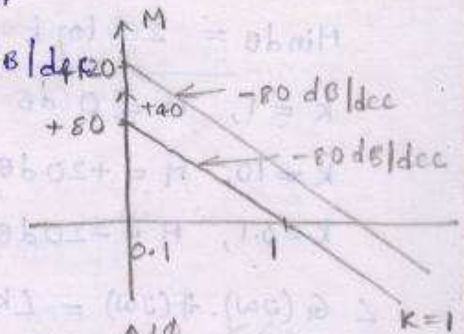
$= -80$ dB/decade

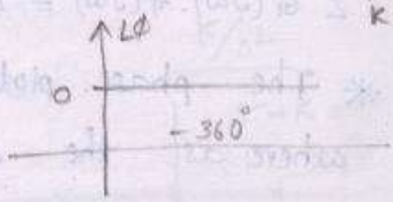
$\angle\phi = -90 \times 4$

$= -360^\circ$

shift $= 20 \log 100$

$= +40$ dB.





Q. $G(s).H(s) = s^n$

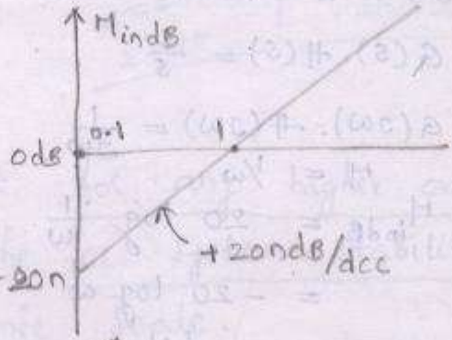
$G(j\omega).H(j\omega) = (j\omega)^n$

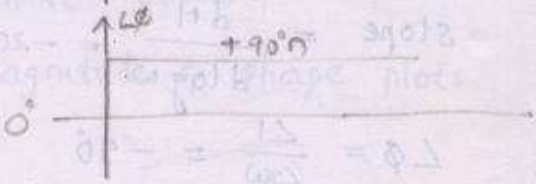
$M = \omega^n$

$M_{in\,dB} = 20 \log \omega$

$\frac{dM}{d\log\omega} = +20n$ dB/dec

$\angle\phi = +90^\circ n$





Q. Draw Bode plot for $\frac{s^7}{10}$

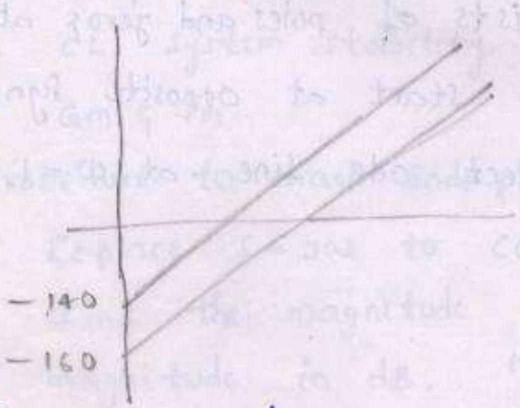
7 zeros at origin.

$\rightarrow +140$

$\rightarrow 90 \times 7 = 630^\circ$

shift $= 20 \log \frac{1}{10}$

$= -20$ dB



Q. $GH = \frac{1}{1+sT}$

$G(j\omega).H(j\omega) = \frac{1}{1+j\omega T}$

$M = \frac{1}{\sqrt{1+(\omega T)^2}}$

$M_{in\,dB\,Actual} = -20 \log \sqrt{1+(\omega T)^2}$; $\phi_{Actual} = -\tan^{-1}(\omega T)$

Asymptotic / Approx.

case 1: $\omega T < 1$ ($\omega < 1/T$), neglect ωT

$M = 0\,dB$, slope $= 0$

$\angle\phi = \frac{\angle 1}{\angle 1} = 0^\circ$

case 2: $\omega T > 1$ ($\omega > \frac{1}{T}$), neglect 1

$M_{asy} = -20 \log(\omega T)$

$\frac{dM}{d\log\omega} = -20\,dB/dec$

$\phi_{asy} = \frac{\angle 1}{\angle j\omega T} = -90^\circ$

The frequency at which slopes changes from one level to another level.

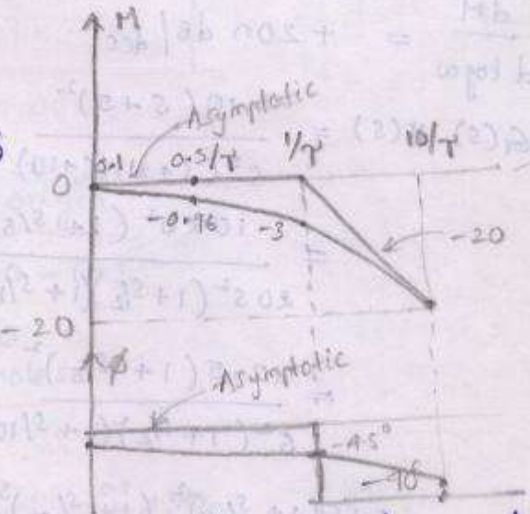
The corner freq's are nothing but a finite poles and finite zeros in the magnitude form.

For one finite poles

	S	ϕ
$< cf \rightarrow$	0	0
$> cf \rightarrow$	$-20\,dB/dec$	-90°

for 'n' finite poles

	S	ϕ
$< cf$	0	0
$> cf$	$-20n\,dB/dec$	$-90^\circ n$



Error at corner frequency :-

Error is nothing but a difference b/w actual and asymptotic value.

$\omega T = 1$, at $\omega = \frac{1}{T}$, $M_{asy} = 0\,dB$

$M_{actual} = -20 \log\sqrt{1+(\omega T)^2} = -20\log\sqrt{2}$

$= -3\,dB$

$E = 3\,dB$

$M_{asy}\left(\omega = \frac{0.5}{T}\right) = 0\,dB$

$M_{act} = -20\log\sqrt{1+0.5^2} = -0.96\,dB$; $E = 0.96\,dB$

Error is maximum at corner freq. On either side of cf, the error decreases symmetrically

$\phi_{act} = -\tan^{-1}\omega T$

At $\omega = \frac{1}{T}$, $\phi_{act} = -\tan^{-1} 1 = -45^\circ$; $\phi_{asy} = 0^\circ$ or -90°

$E = 45^\circ$

$\Rightarrow G(s).H(s) = (1+sT)^n$ $\qquad \phi = \angle j\omega \ldots n$ times

$M_{in\,dB} = +20n \log\sqrt{1+(\omega T)^2}$ $\qquad = 90°$

$\phi_{act} = +n.\tan^{-1}(\omega T)$ $\qquad$ for n- finite zeros

	S	ϕ
$<$ CF $\Rightarrow$	0	0
$>$ CF $\Rightarrow$	+20n dB/dec	+90n

case 1: $\omega T < 1$, neglect ωT

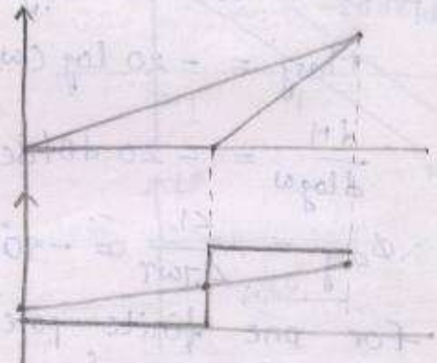
$M_{asy} = 0, \ \phi_{asy} = 0$

case 2: $\omega T > 1$, neglect 1,

$M_{asy} = +20n \log \omega T$

$= +20n \log\omega + \cancel{20n \log T}^{0}$

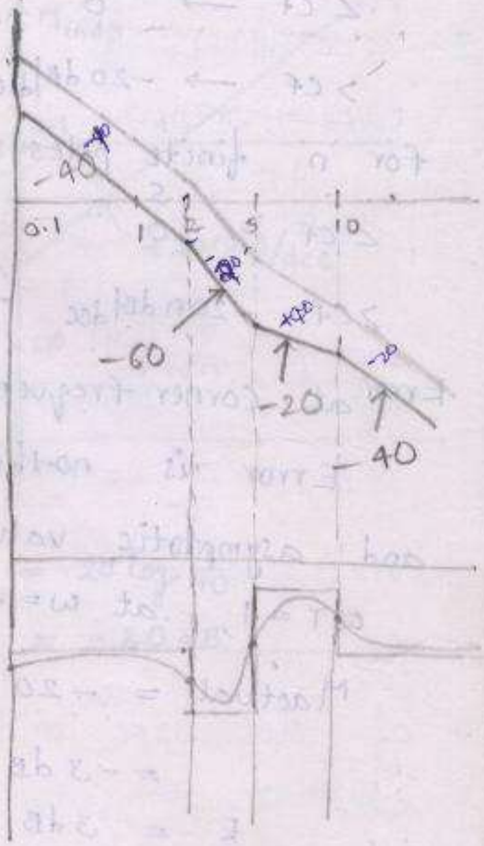
$$\frac{dM}{d\log\omega} = +20n \text{ dB/dec}$$



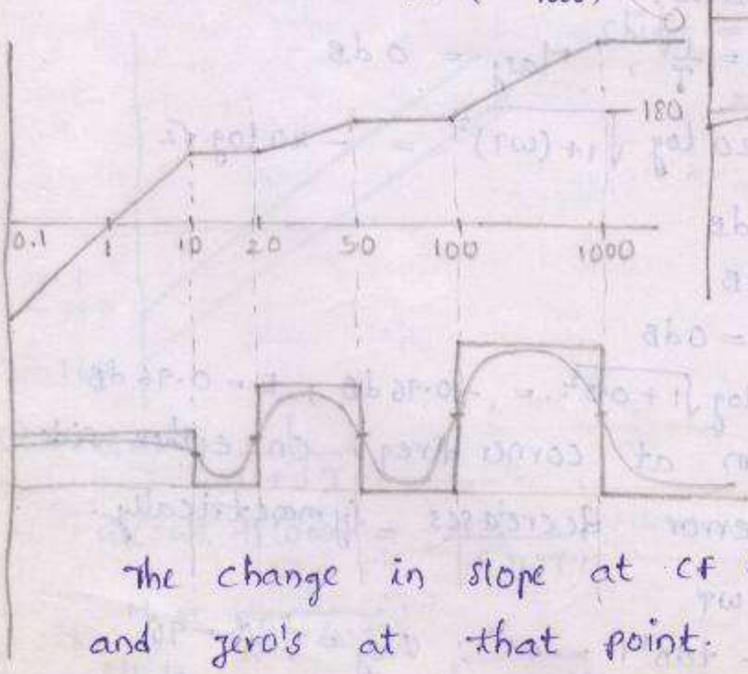
Q. $G(s).H(s) = \dfrac{10(s+5)^2}{s^2(s+2)(s+10)}$

$= \dfrac{10\times 5^2(1+s/5)^2}{20 s^2(1+s/2)(1+s/10)}$

$= \dfrac{12.5(1+s/5)^2}{s^2(1+s/2)(1+s/10)}$



Q. $G(s)H(s) = \dfrac{\overset{1.0}{\cancel{0.1}} s(1+s/20)^2 \cdot (1+s/100)^3}{(1+s/10)(1+\frac{s}{50})^2(1+\frac{s}{1000})^3}$



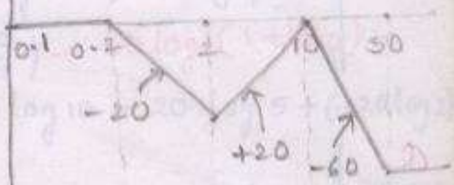
The change in slope at CF is nothing but poles and zero's at that point.

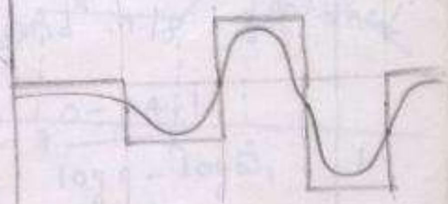
Q. $G(s) \cdot H(s) = \dfrac{1 \cdot (1 + s/2)^2 (1 + s/50)^3}{(1 + s/0.2)(1 + s/10)^4}$

The

NOTE :-

The change in slope at corner frequency is nothing but poles & zeros at that point.





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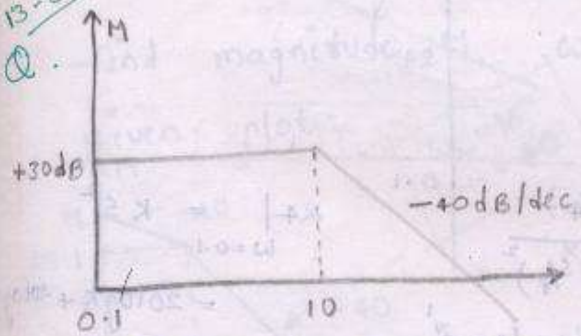
Q.



Initial slope → P/Z → origin

change in slope = $-40 - (0)$

$= -40$ → poles

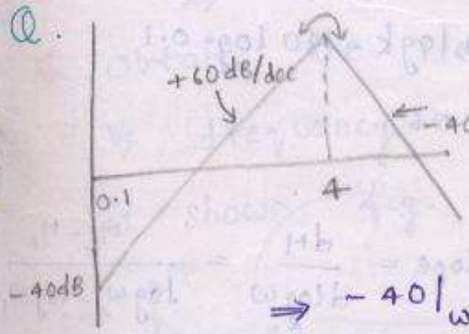
$$30\Big|_{\omega=0.1} = \frac{k}{(1+s/10)^2}$$

$$\Rightarrow 30 = 20 \log k - 40 \log (1 + s/10)$$

Refer this once again

$$\Rightarrow 30 = 20 \log k \Rightarrow k = 10^{1.5} =$$

Q.



$$\frac{k s^3}{(1 + s/4)^5} = -40\Big|_{\omega=0.1}$$

Change in slope $= -40 - (+60)$

$= -100$ dB/dec. → 5 poles

$$\Rightarrow -40\Big|_{\omega=0.1} = 20 \log k + 60 \log 0.1$$

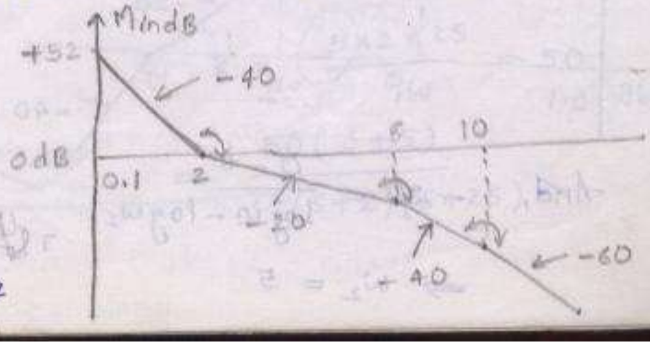
$$\Rightarrow k = 10$$

Q.

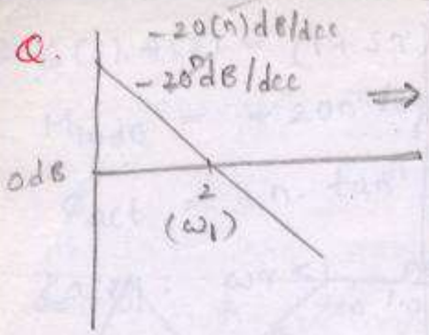
$$\frac{k(1 + s/2)}{s^2 (1 + \frac{s}{8})(1 + s/10)}$$

$$0\Big|_{\omega=2} = 20 \log k - 40 \log 2$$

$$\Rightarrow k = 4$$

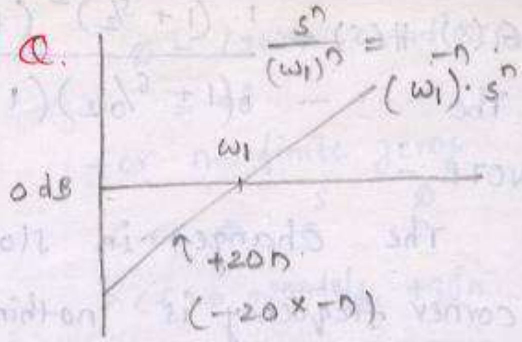


Q.



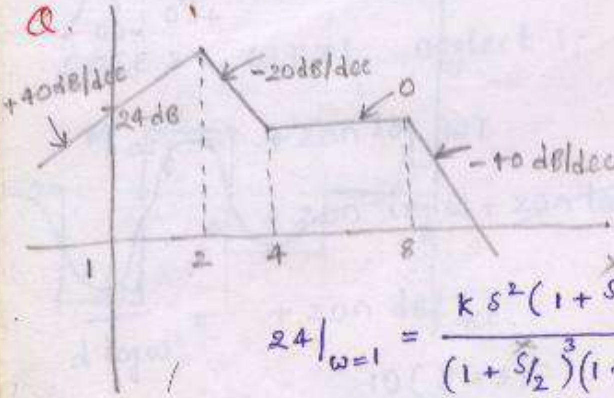
$\Rightarrow \dfrac{(\omega_1)^n}{s^n}$

Q.



$\dfrac{s^n}{(\omega_1)^n} = (\omega_1)^{-n} \cdot s^n$

Q.

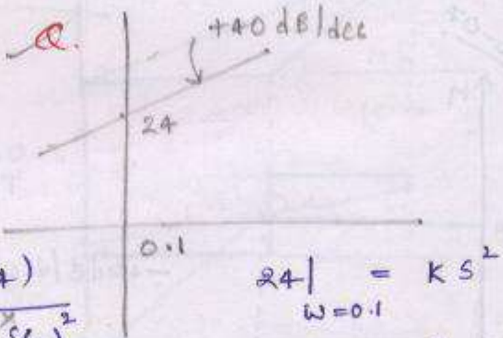


$$24\Big|_{\omega=1} = \frac{k s^2 (1 + s/4)}{(1 + s/2)^3 (1 + s/8)^2}$$

$$24 = 20 \log k + 40 \log 1$$

$$\Rightarrow k = 15.8$$

Q.

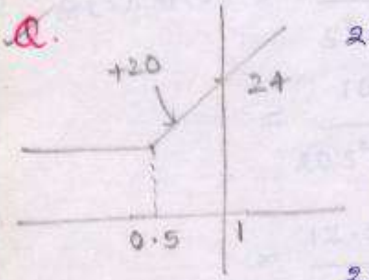


$$24\Big|_{\omega=0.1} = k s^2$$

$$= 20 \log k + 40 \log 0.1$$

$$\Rightarrow k = 1584.8$$

Q.

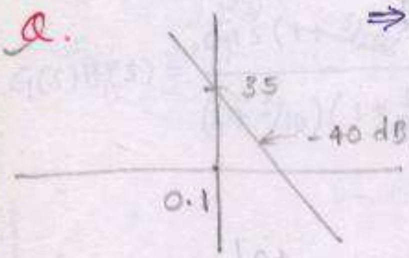


$$24\Big|_{\omega=1} = k(1 + s/0.5)$$

$$24 = 20 \log k + 20 \log\left(1 + \frac{1}{0.5}\right)$$

$$\Rightarrow k = 7.9$$

Q.

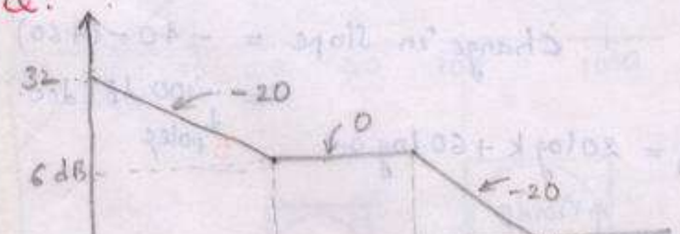


$$35\Big|_{\omega=0.1} = \frac{k}{s^2}$$

$$35 = 20 \log k - 40 \log 0.1$$

$$\Rightarrow k = 0.56.$$

Q.



$$\text{slope} = \frac{dM}{d\log\omega} = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

$$\Rightarrow -20 = \frac{6 - 32}{\log\omega_1 - \log 0.1}$$

$$\Rightarrow \omega_1 = 2$$

And, $-20 = \dfrac{0 - 6}{\log 10 - \log\omega_2}$

$$\Rightarrow \omega_2 = 5$$

$$T/F = \frac{k(1 + s/2)}{s(1 + s/5)(1 + s/10)}$$

check, $32 /_{0.1} = \dfrac{k(1+s/2)}{s(1+s/5)(1+s/10)}$

also for $6/_{\omega=2}$

$6/\omega=5$

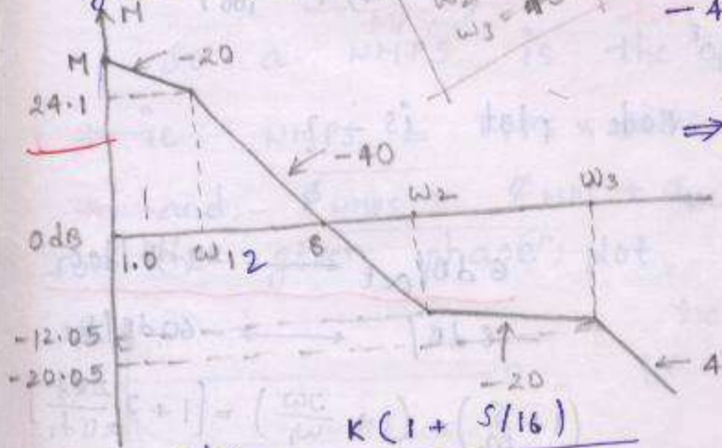
$0/\omega=10 = 20\log k - 20\log 10 + 20\log(1+\frac{10}{2})$
$-20\log(1+\frac{10}{5}) - 20\log(1+10/10)$

$0 = 20\log k - 20\log 10 + 20\log 5 + (-20\log 2)$

$\Rightarrow k = 4$

Q. find magnitude M, $\omega_1, \omega_2, \omega_3$ and T/f for the given plot.

$\omega_2 = 16$
$\omega_3 = 40$



$-40 = \dfrac{0-24.1}{\log 8 - \log \omega_1}$

$\Rightarrow \omega_1 = 2$ rad/sec.

$-20 = \dfrac{24.1 - M}{\log 2 - \log 1}$

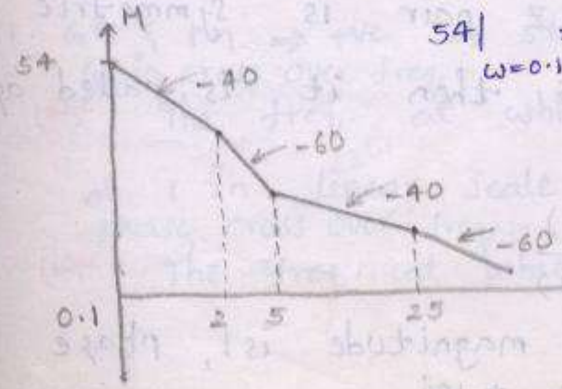
$\Rightarrow M = 30.12$ dB.

$T/f = \dfrac{k(1+s/16)}{s(1+s/2)(\frac{s}{40}+1)}$

$30.12|_{\omega=1} = 20\log k - 20\log 1$

$\Rightarrow k = 32$

Q. Obtain the T/f for the given log magnitude vs frequency plot of a min. phase system is shown fig.

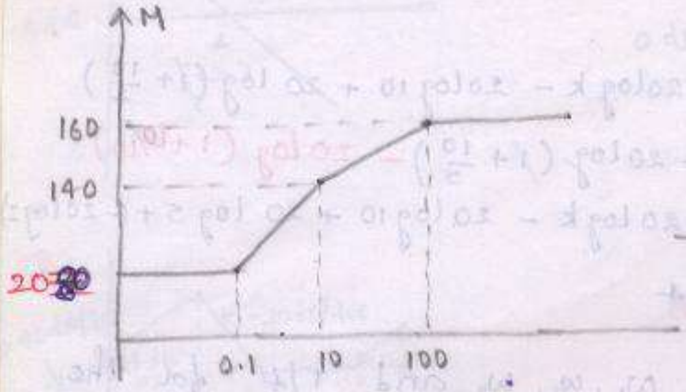


$54|_{\omega=0.1} = \dfrac{k(1+s/5)}{s^2(1+s/2)(1+s/25)}$

$?\ k = \dfrac{5\times 2\times 25}{5} = 50$

$= \dfrac{50(s+5)}{s^2(s+2)(s+25)}$

Q. The approx. Bode plot of a min. ph. system shown in fig. A. T/f of the system is — ?



2 dec ⟶ 120

1 dec ⟶ 60 dB/dec

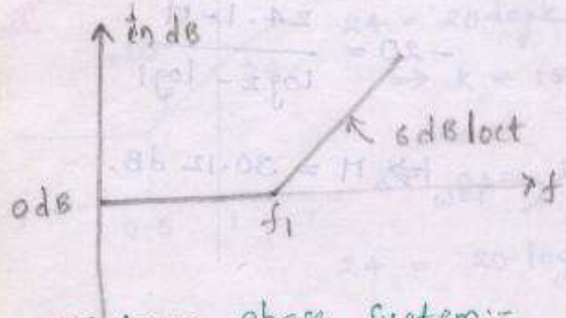
⟶ for 0.1 to 10 ⟶ +60 dB/dec

10 - 100 ⟶ +20 dB/dec

$$20\Big|_{\omega=0.1} = \frac{K\left(\frac{s}{0.1}+1\right)^3}{\left(1+s/10\right)^2\left(1+\frac{s}{100}\right)}$$

$$K' = \frac{10\times 10^2 \times 100}{(0.1)^3} = 10^8$$

Q. The fun. corr. to. Bode plot is — ?



6 dB/oct ⟷ 20 dB/dec

−18 dB/ ⟷ −60 dB/dec

$$\left(1+\frac{s}{\omega_1}\right) = \left(1+\frac{j\omega}{\omega_1}\right) = \left[1+j\frac{2\pi f}{2\pi f_1}\right]$$
$$= \left[1+j\,f/f_1\right]$$

Minimum phase system:-

A system in which all the poles and zeros in the LHS then it is called min. phase system.

Ex: $\dfrac{(s+1)}{(s+2)(s+3)}$

ALPHAS system:-

A system in which zeros lies on Right of s-plane, poles lies on the left of s-plane and the loc. of p-z pair is symmetric about imaginary axis then it is called as Alpha system.

Ex: $\dfrac{(s-2)(s^2-2s+2)}{(s+2)(s^2+2s+2)}$

* for Alpha system magnitude is 1, phase angle ± 180°.

The control systems are low pass system.

Non-minimum phase system:-

A system in which one & more z's located in right side of s-plane and remain all p's & z's are in LHS then it is called a Non-minimum phase system.

$$Ex:- \frac{(s-1)(s+4)}{(s+2)(s+3)}$$

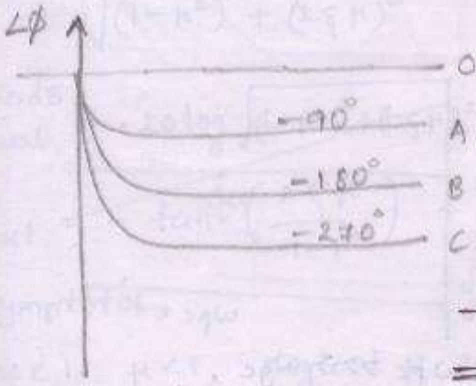
$$= \underbrace{\frac{(s+1)(s+4)}{(s+2)(s+3)}}_{MPS} \cdot \underbrace{\frac{(s-1)}{(s+1)}}_{ALPS}$$

So a NMPS is the product of MPS & ALPS

* ie NMPS = MPS × ALPS

* and $\phi_{NMPS} = \phi_{MPS} + \phi_{ALPS}$

Q. for the given phase plot, the A, B, C plots are –



A: MPS

B: ALPS

C: NMPS

$$-90° - 180°$$
$$= -270°$$

Stability Conditions :- → To find c/L system stability.

Gain margin $GM = \frac{1}{M}\Big|_{\omega=\omega_{pc}}$

c/L system stability given by Char. eq : ie $1+G(s)H(s)=0$

phase margin $PM = 180 + \angle GH\big|_{\omega=\omega_{gc}}$

1. GM & PM ⇒ +ve → stable. ; $\omega_{pc} > \omega_{gc}$, $GM > 1$

Gain cross over freq :-

(ω_{gc}) The freq. at which the magnitude = 0 dB or 1 in linear scale.

phase cross over freq :-

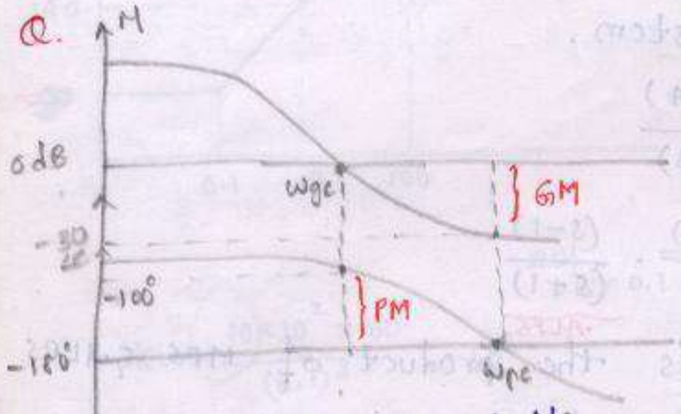
(ω_{pc}) The freq. at which ph. angle is $-180°$.

2. $\omega_{pc} = \omega_{gc} \longrightarrow GM = PM = 0,\ \longrightarrow$ m.s.

$GM = 1$

3. $\omega_{pc} < \omega_{gc} \Rightarrow GM$ -ve, <1, PM -ve $\Big\}$ unstable.

Q.



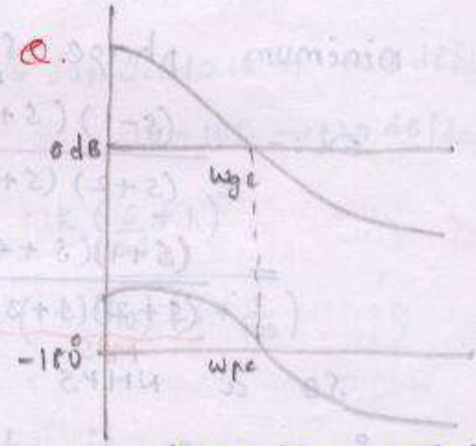
$\omega_{pc} > \omega_{gc} \longrightarrow$ stable

$GM = -20\log(M)_{\omega=\omega_{pc}}$

$GM = -(-30dB) = 30dB$

$PM = 180 - 100 = 80$

Q.

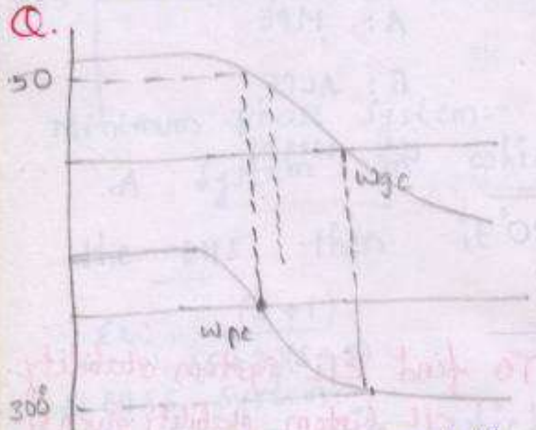


$\omega_{gc} = \omega_{pc} \longrightarrow$ m.s

$GM = -(0dB) = 0\,dB$

$PM = 180 - 180 = 0$

Q.

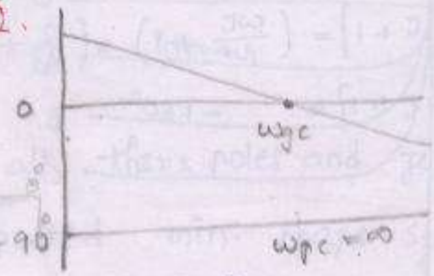


$\omega_{pc} < \omega_{gc} \longrightarrow$ unstable

$GM = -(+50) = -50\,dB$

$PM = 180 - 300 = -120°$

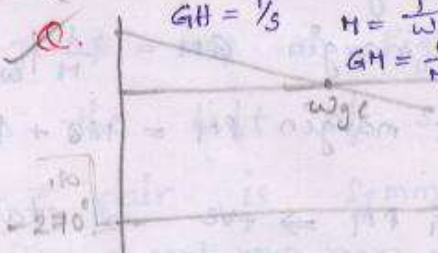
Q.



$\omega_{pc} >>> \omega_{gc}$,

Absolutely stable.

Q.



$GH = 1/s$ $\quad M = \frac{1}{\omega}\Big|_{\omega=\omega_{pc\infty}} = 0$

$GM = \frac{1}{M}\Big|_{\omega=\omega_{pc}} = \infty$

$PM = 180 - 90 = 90°$.

~~$\omega_{pc} < \omega_{gc}$~~

$\omega_{pc} << \omega_{gc} \rightarrow$ unstable

$\omega_{pc} = 0$

$PM = 180 - 270 = -90$

BODE PLOTS FOR COMPLEX P/Z's :-

complex poles

$$GH = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$0 \le \xi \le 1$

$$= \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n j\omega + \omega_n^2}$$

$$= \frac{1}{-\frac{\omega^2}{\omega_n^2} + j2\xi\frac{\omega}{\omega_n} + 1}$$

$$= \frac{1}{1 - \frac{\omega^2}{\omega_n^2} + 2j\xi\frac{\omega}{\omega_n}}$$

$$= \frac{1}{(1-\mu^2) + j2\xi\mu}$$

$$M = \frac{1}{\sqrt{(1-\mu^2)^2 + (2\xi\mu)^2}}$$

$$M_{in\,dB}\ \text{Actual} = -20\log\sqrt{(1-\mu^2)^2 + (2\xi\mu)^2}$$

$$\phi_{act} = -\tan^{-1}\left(\frac{2\xi\mu}{1-\mu^2}\right)$$

Asymptotic,

case 1: $\mu < 1$, neglect μ, $2\xi\mu$

$$M_{asy} = -20\log 1$$
$$= 0$$

$$\phi_{asy} = 0$$

case 2: $\mu > 1$, neglect 1,

$$M_{asy} = -20\log\sqrt{\mu^4}$$
$$= -40\log\frac{\omega}{\omega_n}$$
$$= -40\log\omega + 40\log\omega_n$$

$$\text{slope} = \frac{dM}{d\log\omega} = -40\ \text{dB/dec}$$

complex zeros

$$\frac{s^2 + 2\xi\omega_n s + \omega_n^2}{\omega_n^2}$$

$$= \frac{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2}{\omega_n^2}$$

$$= -\frac{\omega^2}{\omega_n^2} + j2\xi\frac{\omega}{\omega_n} + 1$$

$$= \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + j\left[2\xi\frac{\omega}{\omega_n}\right]$$

$$= (1-\mu^2) + (2\xi\mu)j$$

let $\mu = \frac{\omega}{\omega_n}$

$$M = \sqrt{(1-\mu^2)^2 + (2\xi\mu)^2}$$

$$M_{in\,dB}\ \text{Act.} = 20\log\sqrt{(1-\mu^2)^2 + (2\xi\mu)^2}$$

$$\phi_{act} = \tan^{-1}\left(\frac{2\xi\mu}{1-\mu^2}\right)$$

$$M_{asy} = 0\ \text{dB}$$

$$\phi_{asy} = 0$$

$$M_{asy} = +40\log\mu^{\,\omega/\omega_n}$$

$$\text{slope} = +40\ \text{dB/decay}$$

$\phi_{asy} = -\tan^{-1}\left(\frac{\overbrace{2\xi\mu}^{\text{very small}}}{\underbrace{1-\mu^2}_{\text{neglect}}}\right)$ $\qquad$ $\phi_{asy} = +180°$

$= -\tan^{-1}(-0 \text{ very small})$

$= -(180 - \tan^{-1}0)$

$= -180°$

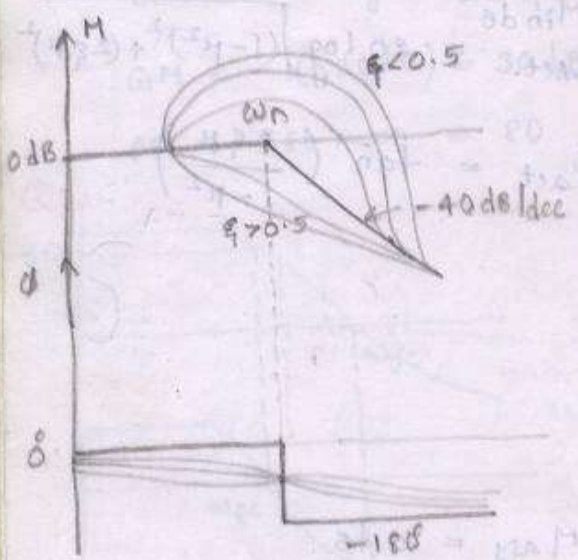
$< cf$	0	0
$> cf$	$-40\ dB/dec$	$-180°$

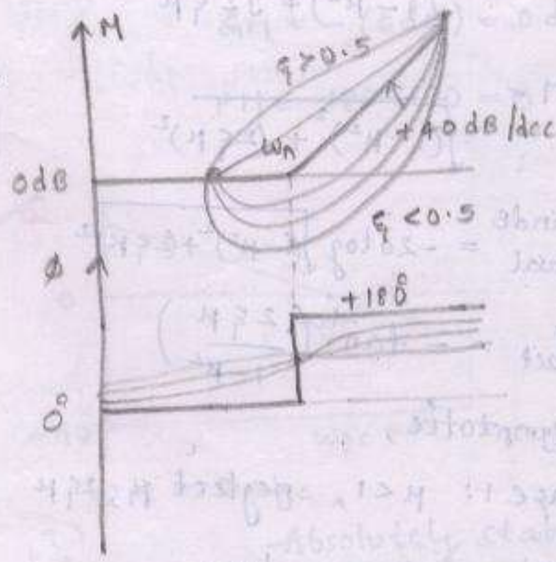
$< cf$	0	0
$> cf$	$+40\ dB/dec$	$+180$

for n- complex poles

$< cf$	0	0
$> cf$	$-40n\ dB/dec$	$-180°n$

$< cf$	0	0
$> cf$	$+40n\ dB/dec$	$+180°n$





correction of cf :-

$M_{act} = -20\log\sqrt{(1-\mu^2)^2 + (2\xi\mu)^2}$

$\omega = \omega_n$

$\Rightarrow \mu = 1 \quad M_{correction} = -20\log 2\xi$

$\xi = 0.1, \quad M = -20\log 0.2 =$

$\xi = 0.8, \quad M = -20\log 1.6 =$

$\phi = -\tan^{-1}\left(\frac{2\xi\mu}{1-\mu^2}\right)$

$= -90$

for 'n' no. of ,

$M_{correction} = -20n \log 2\xi$ $\qquad$ $M_{corr.} = 20n \log 2\xi$

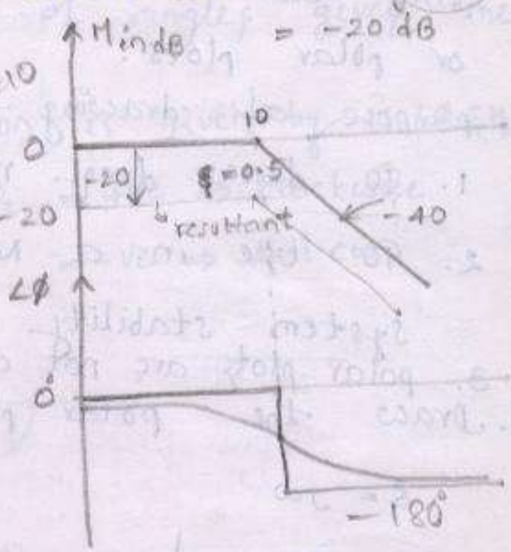
Q. Draw the Bode plot for $\dfrac{10}{s^2+10s+100}$

$$GH(s) = 0.1\left[\frac{100}{s^2+10s+100}\right]$$

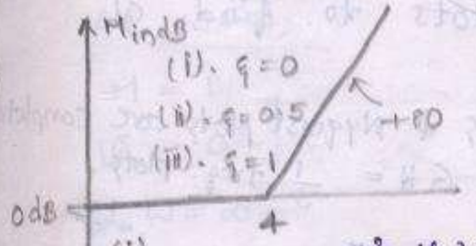
Shift $= 20\log 0.1 = -20\text{ dB}$

Refer once again

$\omega_n = 10$



Q. $GH(s) = \dfrac{\quad}{} ?$



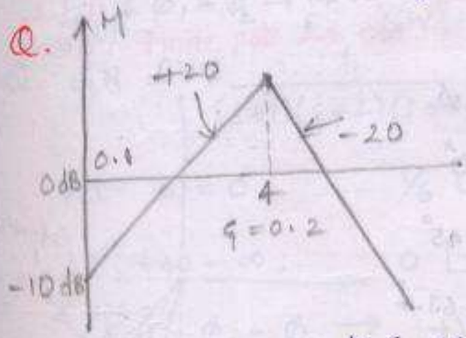
(i). T/f : $k\left[\dfrac{s^2+16}{4}\right]^2$

$0|_{\omega=4} = 20\log k$

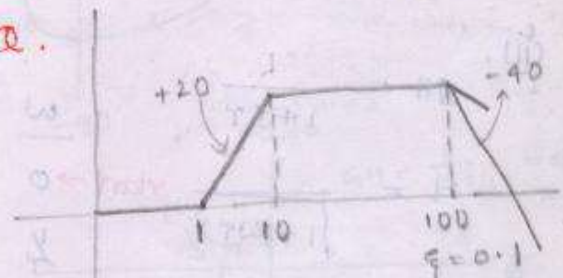
$\Rightarrow k = 1$

(ii). T/f : $\dfrac{(s^2+4s+16)^2}{16}$ $\qquad$ (iii). T/f : $\dfrac{(s^2+8s+16)^2}{16}$

Q.



Q.



$$-10|_{\omega=0.1} = \frac{ks\ 16}{s^2+16s+16}$$

$$0|_{\omega=1} = \frac{k(1+s/1).10^4}{(1+s/10).(s^2+20s+10^4)}$$

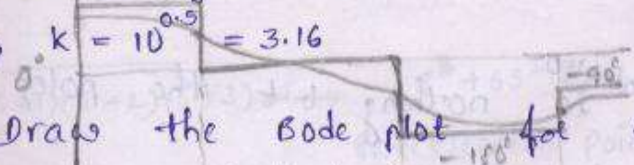
$-10 = 20\log k + 20\log 0.1$

$10 = 20\log k$

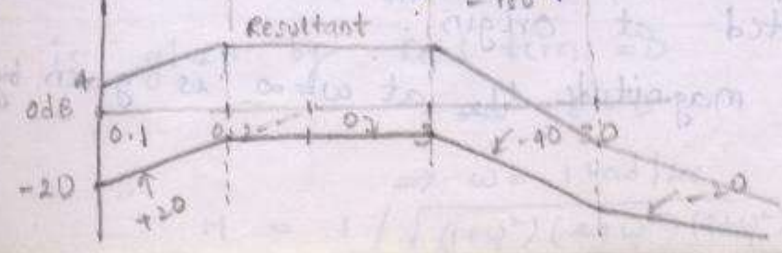
$\Rightarrow k = 10^{0.5} = 3.16$

$0 = 20\log k$

$\Rightarrow k = 1$



Q. Draw the Bode plot for $G(s)H(s) = \dfrac{16s(1+s/30)}{(1+s/0.2)(1+\frac{s}{3}+\frac{s^2}{9})}$



Shift $= 20\log 16 = 24\text{ dB}$

Limitation of Bode plot :-

Drawing 2 plots to find the CL system stability. This can be avoided by drawing Nyquist plots or polar plots.

Purpose of drawing polar plot :-

1. To draw freq. response of OL T/f.
2. To use in a Nyquist plots to find CL system stability.
3. polar plots are not complete plots & Nyquist plots are complete freq. plots.

Q. Draw the polar plot for $GH = \frac{1}{s}$

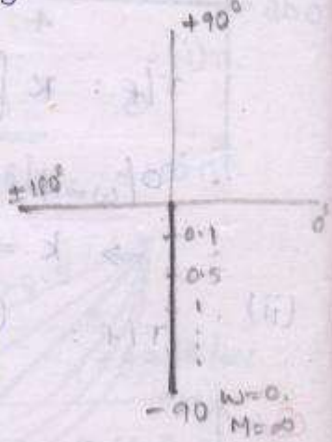
$s = j\omega$

$\Rightarrow GH = \frac{1}{j\omega}$

$M = \frac{1}{\omega}$

$L\phi = \frac{\angle 1}{\angle j\omega} = -90^\circ$

ω	M	Lφ
start 0	∞	−90°
1	1	−90°
2	0.5	−90°
10	0.1	⋮
⋮	⋮	⋮
∞ end	0	−90°



(ii).

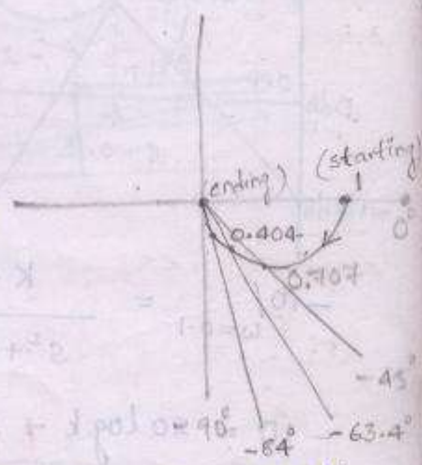
$GH = \frac{1}{1+sT}$

$M = \frac{1}{\sqrt{1+(\omega T)^2}}$

$\angle\phi = \frac{\angle 1}{\angle(1+j\omega T)}$

$= -\tan^{-1}(\omega T)$

ω	M	∠φ
start → 0	1	0°
1/T	0.707	−45°
2/T	0.404	−63.4°
10/T	0.1	−84°
⋮		
end → ∞	0	−90°



* At $\omega = 0$, the magnitude M_1 is given by substituting $s = 0$.

* The ph. angle ϕ_1 at $\omega=0$ is nothing but the poles and zero's located at origin.

* The ending magnitude M_2 at $\omega = \infty$ is given by sub. $s = \infty$.

* for ending phase angle at $s=\infty$, consider the $-90°$ for each pole and $+90°$ for each zero. The algebraic sum of angles gives the ending angle.

* Direction is given by $M_1\angle\phi_1$ & $M_2\angle\phi_2$

$\phi_1 - \phi_2 = +ve \Rightarrow$ clockwise

$= -ve \Rightarrow$ Anti cw.

Q. $GH = \dfrac{1}{s+1}$

At $\omega = 0$,

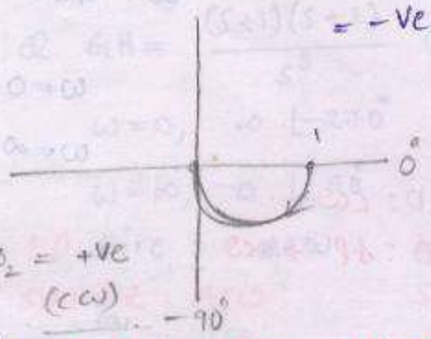
$M = 1$

$\angle\phi = 0°$

At $\omega = \infty$,

$0 \angle -90°$

$\phi_1 - \phi_2 = +ve$ (cw)



Q. $GH = \dfrac{1}{(s+1)(s+2)}$

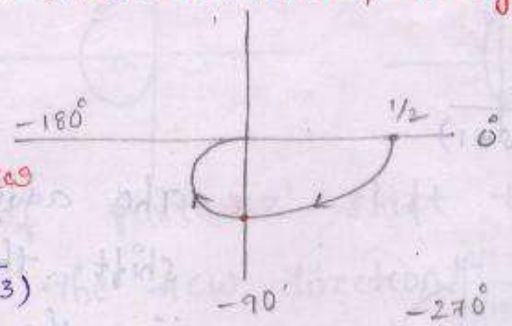
* Intersection point is nothing but magnitude

At $\omega = 0°$; $\frac{1}{2}\angle 0°$

At $\omega = \infty$, $0 \angle -180°$

ED: $\phi_1 - \phi_2 \rightarrow +ve \rightarrow$ cw (cw)

SD: finite pole $\rightarrow$ cw



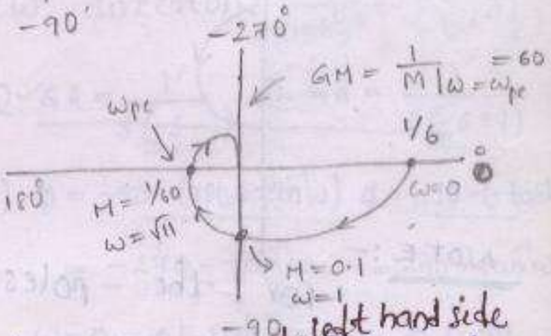
Q. $GH = \dfrac{1}{(s+1)(s+2)(s+3)}$

At $\omega = 0°$; $\frac{1}{6}\angle 0°$

At $\omega = \infty$; $0 \angle -270°$

ED: $\phi_1 - \phi_2 \rightarrow +ve \rightarrow$ cw' (cw)

SD: finite pole $\rightarrow$ cw.



* The addition of each finite pole in the left hand side shift the ending angle by $-90°$ in the cw direction.

$$\frac{1}{(s+1)(s+2)(s+3)} \longrightarrow s^3 + 6s^2 + 11s + 6$$

Intersection point with ima. axis is given by Real terms $= 0$

$$-6\omega^2 + 6 = 0$$

$$\Rightarrow \omega = 1 \text{ rad/sec}$$

$$M = 1 \Big/ \sqrt{(1+\omega_1^2)(4+\omega_1^2)(9+\omega_1^2)} = \frac{1}{\sqrt{2\times5\times10}} = \frac{1}{10}.$$

Intersection point with Real axis $\Rightarrow$ Ima. part = 0

$\Rightarrow -jw^3 + 11jw = 0$ Intersection point with

$\Rightarrow \omega = \sqrt{11}$ rad/sec   −ve real axis = ω_{pc}.

$$M = \frac{1}{\sqrt{12 \times 15 \times 20}} = \frac{1}{60}$$

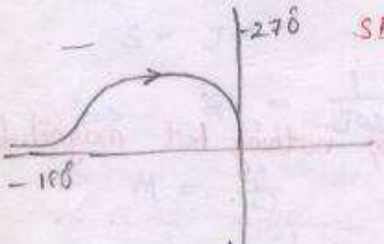
Q. $GH = \dfrac{1}{s^2(s+1)}$

$\omega = 0;\ \infty \angle -180^\circ$

$\omega = \infty;\ 0 \angle -270^\circ$

$\overline{\text{cw}}$ ED: CW

SD: 4P → CW



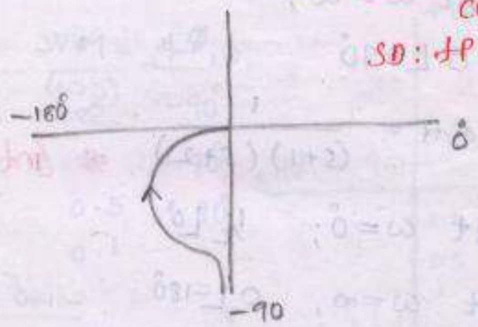
Q. $GH = \dfrac{1}{s(s+1)}$

$\omega = 0;\ \infty \angle -90^\circ$

$\omega = \infty;\ 0 \angle -180^\circ$

$\overline{\text{cw}}$ ED: +ve CW

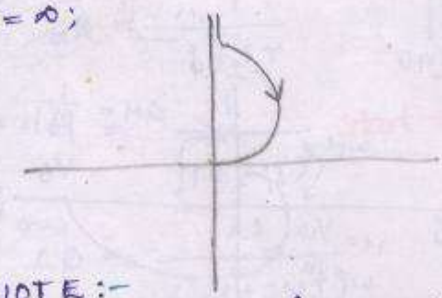
SD: 4P: CW



Q. $GH = \dfrac{1}{s^3(s+1)}$

$\omega = 0;$

$\omega = \infty;$



* The addition of pole at origin shift the total plot by -90° in the cw direction.

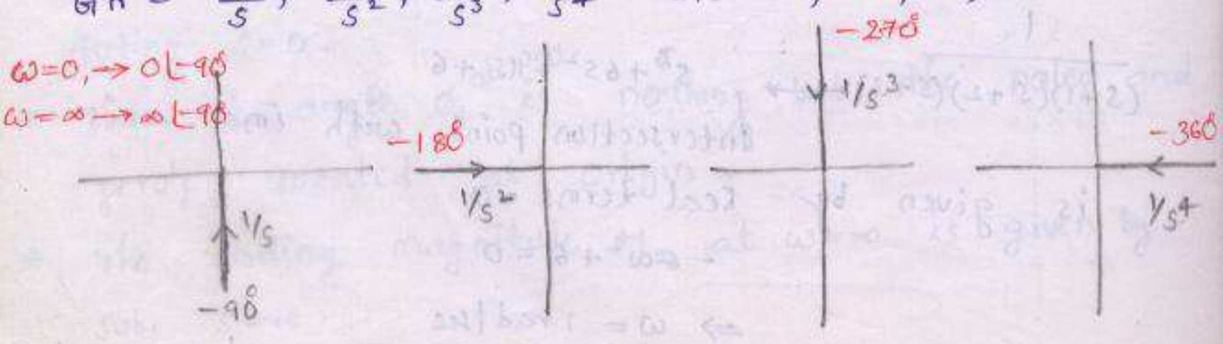
NOTE :- for the poles and z's at origin the polar plot is nothing but a angle lines.

[T/F should not consists any finite p's and z's]

$GH = \frac{1}{s}, \frac{1}{s^2}, \frac{1}{s^3}, \frac{1}{s^4}$. and s, s^2, s^3, s^4.

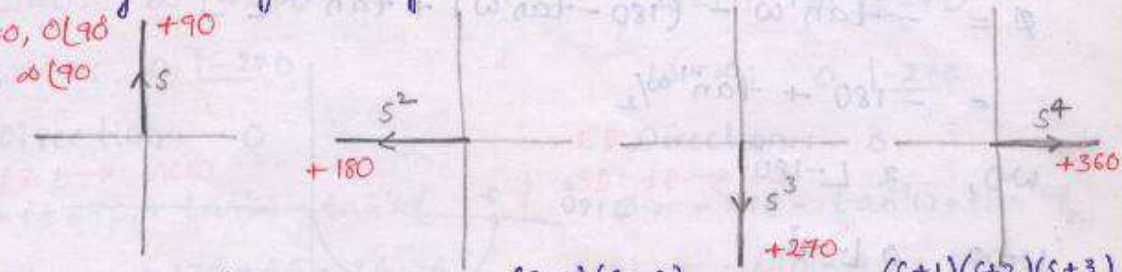
$\omega = 0, \rightarrow 0 \angle -90^\circ$

$\omega = \infty \rightarrow \infty \angle -90^\circ$



* The addition of z at origin shift the (total plot) ending angle by $+90^\circ$ in ACW direction.

$\omega = 0$, $0\angle 90$
$\omega = \infty$, $\infty\angle 90$



Q. $GH = \frac{s+1}{s^3}$

$\omega = 0;\ \infty\angle -270^\circ$

$\omega = \infty,\ 0\angle -180^\circ$

ED: dire : ACW

SD : f Z : ACW

Q. $GH = \frac{(s+1)(s+2)}{s^3}$

$\omega = 0,\ \infty\angle -270^\circ$

$\omega = \infty,\ 0\angle -90^\circ$

ED: Dire : ACW

SD : f Z : ACW

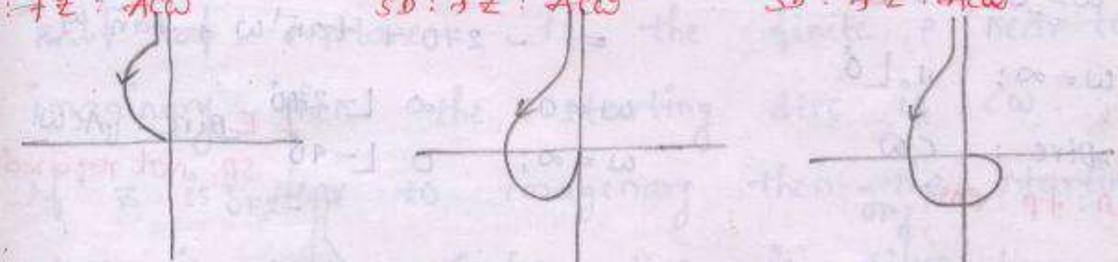
Q. $GH = \frac{(s+1)(s+2)(s+3)}{s^3}$

$\omega = 0,\ \infty\angle -270^\circ$

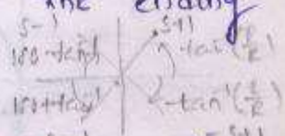
$\omega = \infty,\ 1\angle 0^\circ$

ED: Dire : ACW

SD : f Z : ACW



* The addition of finite z' shift the ending angle by 90° in the ACW direction.



Q. $GH = \frac{1}{s(s+1)}$

$\phi = -90 - \tan^{-1}\omega$

$\omega = 0,\ \infty\angle -90^\circ$

$\omega = \infty,\ 0\angle -180^\circ$

E.Dire : CW

Q. $GH = \frac{1}{s(s-1)}$

$\phi = -90 - (180 - \tan^{-1}\omega)$

$= -270 + \tan^{-1}\omega$

$\omega = 0,\ \infty\angle -270^\circ$

$\omega = \infty,\ 0\angle -180^\circ$

E.Dire : ACW

Q. $GH = \frac{1}{s(-s-1)}$

$\phi = -90 - (180 + \tan^{-1}\omega)$

$= -270 - \tan^{-1}\omega$

$\omega = 0,\ \infty\angle -270^\circ$

$\omega = \infty,\ 0\angle -360^\circ$

E.Dire : CW

Q. $GH = \frac{1}{s(-s+1)}$

$\phi = -90 - (-\tan^{-1}\omega)$

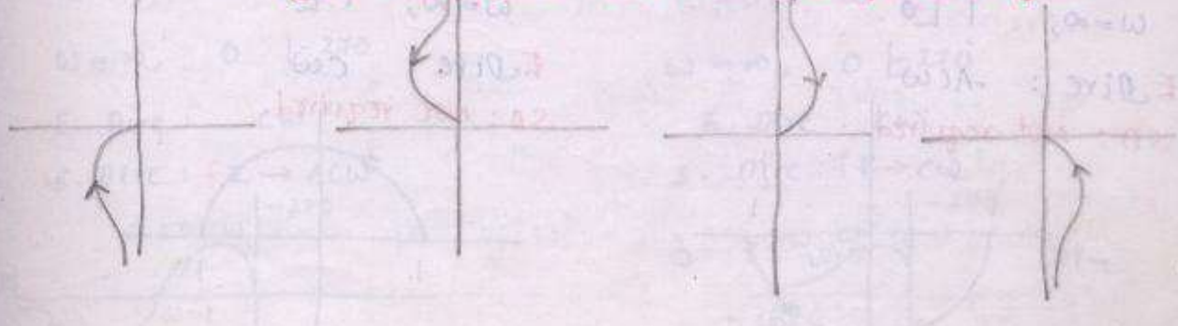
$= -90 + \tan^{-1}\omega$

$\omega = 0,\ \infty\angle -90$

$\omega = \infty,\ 0\angle 0^\circ$

E.Dire : ACW

(SD: Not required b'coz T/F consists -ve sign)



Q. $GH = \frac{(s+2)}{(s+1)(s-1)}$

$\phi = -\tan^{-1}\omega - (180 - \tan^{-1}\omega) + \tan^{-1}\omega/_2$

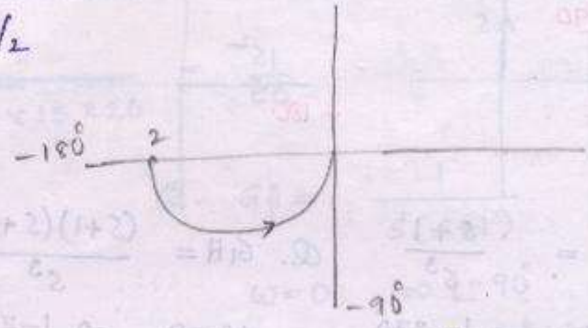
$= -180 + \tan^{-1}\omega/_2$

$\omega = 0$, $2 \angle -180^\circ$

$\omega = \infty$, $0 \angle -90^\circ$

E. Dire : ACW

SD : Not required



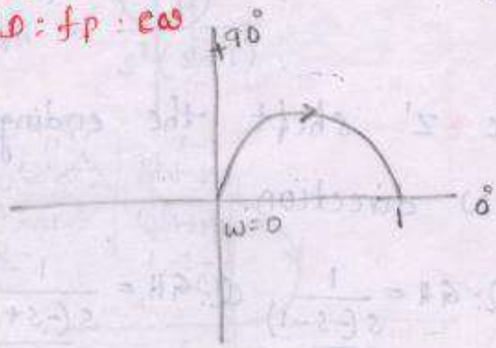
Q. $GH = \frac{s}{s+1}$

$\omega = 0$; $0 \angle +90^\circ$

$\omega = \infty$; $1 \angle 0^\circ$

E. Dire : CW

SD : fp : CW



Q. $GH = \frac{s+3}{s(s-1)}$

$\phi = -90 - (180 - \tan^{-1}\omega) + \tan^{-1}\omega/_3$

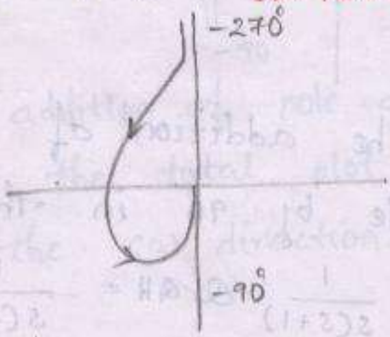
$= -270 + \tan^{-1}\omega + \tan^{-1}\omega/_3$

$\omega = 0$; $\infty \angle -270^\circ$

$\omega = \infty$; $0 \angle -90^\circ$

E. Dire : ACW

SD: Not required



Q. $GH = \frac{s+2}{s-2}$

$\phi = \tan^{-1}\omega/_2 - (180 - \tan^{-1}\omega/_2)$

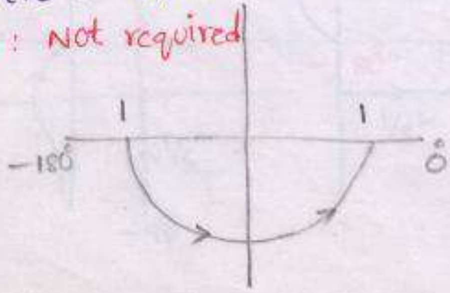
$= -180 + 2\tan^{-1}\omega/_2$

$\omega = 0$, $1 \angle -180^\circ$

$\omega = \infty$, $1 \angle 0^\circ$

E. Dire : ACW

SD : Not required



Q. $GH = \frac{s-2}{s+2}$

$\phi = -\tan^{-1}\omega/_2 + (180 - \tan^{-1}\frac{\omega}{2})$

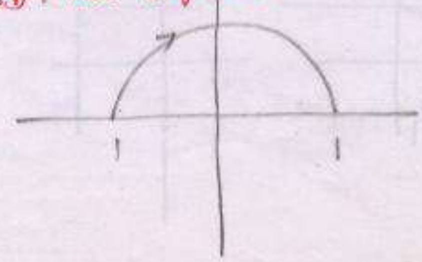
$= 180^\circ - 2\tan^{-1}\omega/_2$

$\omega = 0$; $1 \angle 180^\circ$

$\omega = \infty$, $1 \angle 0^\circ$

E. Dire : CW

SD : Not required.



Q. $GH = \dfrac{s+1}{s^3(s+2)}$

$\omega = 0;\ \infty \angle -270$

$\omega = \infty;\ 0 \angle -270$

ED: Direction : 0

SD: fZ → ACW

$\phi = -270 + \tan^{-1}\omega - \tan^{-1}\omega/2$

$\omega = 1,\ = -270 + 45 - 26.56$

⟹ > -270

Q. $GH = \dfrac{s+2}{s^3(s+1)}$

$\omega = 0;\ \infty \angle -270$

$\omega = \infty;\ 0 \angle -270$

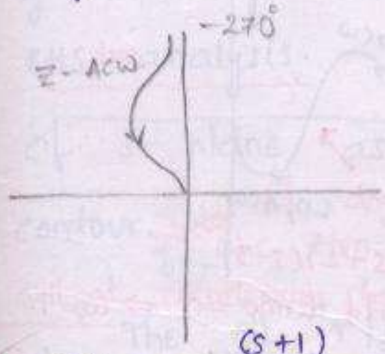
ED: Direction : 0

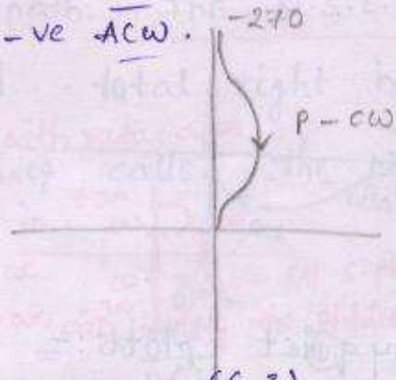
SD: fP → CW

$\phi = -270 - \tan^{-1}\omega + \tan^{-1}\omega/2$

$\omega = 1,\ = -270 - 45 + 26.56$

→ If the T/f consists the finite P and Z's are all in the Left half of s-plane then the starting dire. is given by finite P's and Z's which are left half of s-plane. If the finite P near to imaginary then the starting dire is CW. If Z is near to imaginary then the starting dire is ACW. Ending dire. is given by angle direction $\rightarrow (\phi_1 - \phi_2)$, +ve CW, −ve ACW.





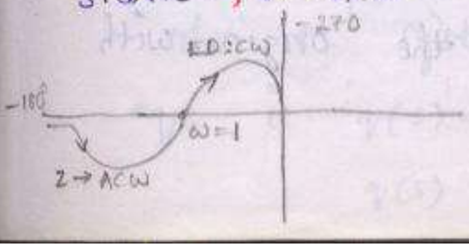
Q. $GH = \dfrac{(s+1)}{s^2(s+2)(s+3)}$

$\omega = 0,\ \infty \angle -180$

$\omega = \infty,\ 0 \angle -270$

E. Dire : CW

S. Dire : fZ → ACW



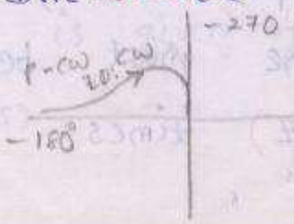
Q. $GH = \dfrac{(s+3)}{s^2(s+1)(s+2)}$

$\omega = 0,\ \infty \angle -180$

$\omega = \infty,\ 0 \angle 270$

E. Dire : CW

S. Dire : fP → CW



$$\frac{(s+1)}{s^2(s+2)(s+3)} \xrightarrow[\text{numerator}]{\text{bring to}} -s^2(2-s)(3-s)(s+1)$$

$$= -s^2(6-5s+s^2)(s+1)$$

$$= -s^5 - 4s^4 - s^3 - 6s^2$$

$\frac{1}{s-2} = \frac{1}{-(2-s)} = -1(2+s)$

Intersection point with real axis:

(odd terms $= 0$) $s^5 - s^3 = 0$

$s \to j\omega$ $\quad -j\omega^5 + j\omega^3 = 0$

$\omega = 1$

verification:

$\phi = -180 - \tan^{-1}\omega/_3 - \tan^{-1}\omega/_2 + \tan^{-1}\omega$

$\angle = -180°$

$$\frac{(s+3)}{s^2(s+1)(s+2)} \longrightarrow -s^2(1-s)(2-s)(s+3)$$

$$= -s^5 + 7s^3 - 6s^2$$

imag. $-s^5 + 7s^3 = 0$

(odd power terms $= 0$)

$s \to j\omega$ $\quad \omega = \pm j\sqrt{7}$ (Invalid point).

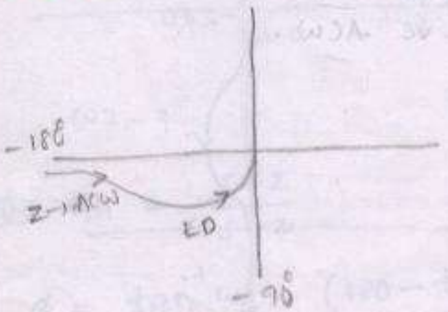
Q. $GH = \frac{(s+1)(s+2)}{s^2(s+3)}$

$\omega = 0$; $\infty \angle{-180°}$

$\omega = \infty$, $0 \angle{-90°}$

E.Dire : ACW

SD: $fZ \to$ ACW



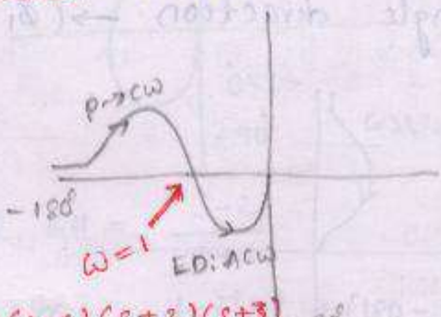
Q. $GH = \frac{(s+2)(s+3)}{s^2(s+1)}$

$\omega = 0$, $\infty \angle{-180°}$

$\omega = \infty$, $0 \angle{-90°}$

E.Dire : ACW

SD: $fP \to$ CW

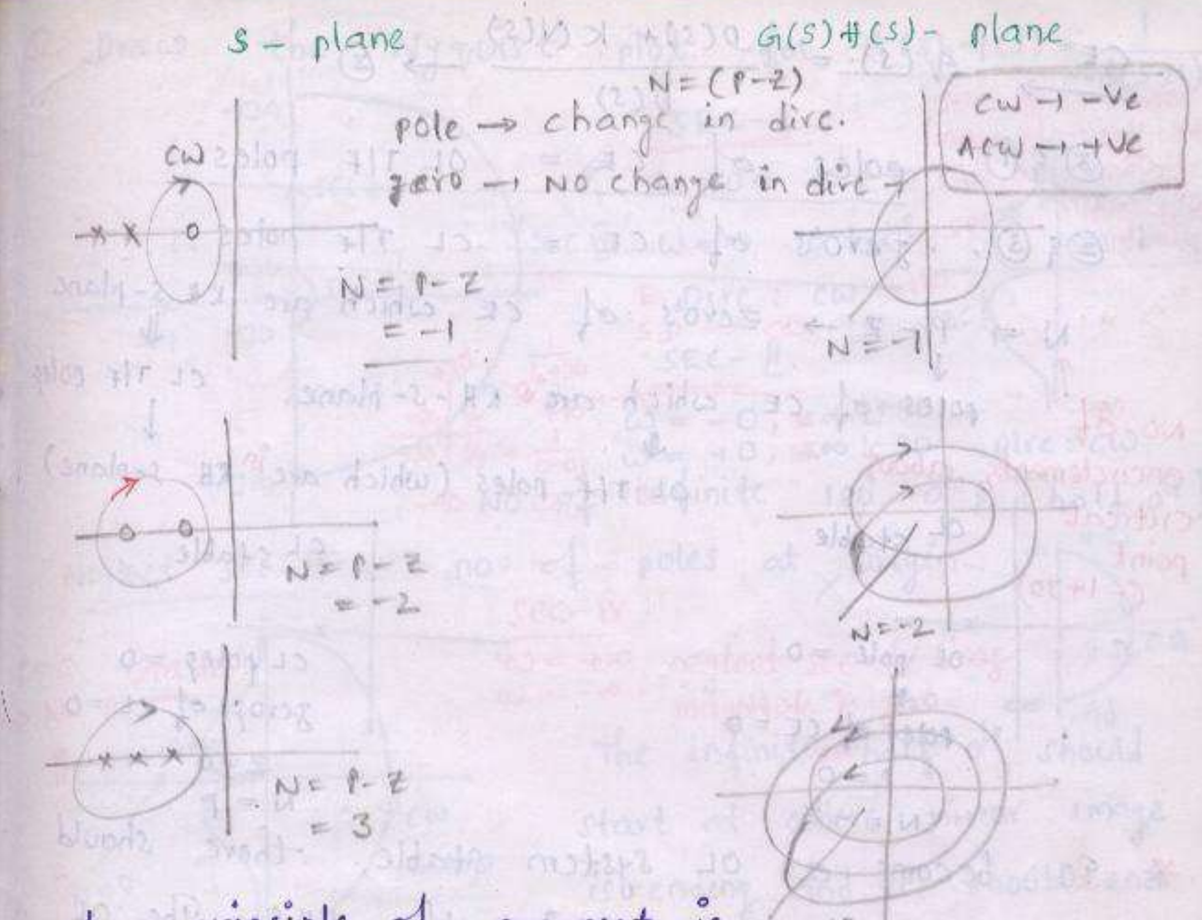


$-s^2(1-s)(s+2)(s+3)$ $\longrightarrow$ Making odd terms $= 0 \Rightarrow \omega = 1$.

Nyquist plots:-

Nyquist stability criteria depends on the principle of arguments:-

principle of arguments states that if there are P poles, Z zeros are enclosed by the s- plane closed path, Then the corr. $G(s)H(s)$ plane must be encircles the origin with $(P-Z)$ times.



The principle of argument is applied to the total right half of s-plane by selecting as a closed path. The N.S.C. is RHS plane analysis. They selected total right half of s-plane as a closed path with radius of ∞ is called the Nyquist contour. +j∞ cw cw cw cw –j∞ If any pole is enclosed in selected N.C. then in G(s)H(s) plane, we RH s-plane will get encirclements, based on encirclements we can check the stability.

The OL T/f $G(S)\,H(S) = K\frac{N(S)}{D(S)} \rightarrow (1)$

P-Z configuration:

CL T/f $G(S)/(H(S)\,G(S)+1) = \frac{C(S)}{R(S)}$

$$= \frac{G(S)}{1 + K\frac{N(S)}{D(S)}}$$

$$\frac{C(S)}{R(S)} = \frac{G(S).\,D(S)}{D(S) + K.N(S)} \rightarrow (2)$$

The CL System stability is given by char. eq. ie $q(S) = 1 + G(S).H(S)$

$$q(S) = 1 + K.\frac{N(S)}{D(S)}$$

$\xrightarrow{CE}$ $q(s) = \dfrac{D(s) + K\,N(s)}{D(s)} \rightarrow$ ③

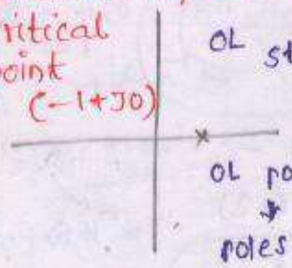
compare ③ & ①. poles of CE = OL T/F poles

②& ③. zero's of CE = CL T/F poles

$N = P - Z$ → zero's of CE which are RH s-plane ⇓ CL T/F poles

↑ No. of encirclements about critical point $(-1+j0)$

↓ poles of CE which are in RH-s-plane ⇓ OL T/F poles (which are in RH s-plane)

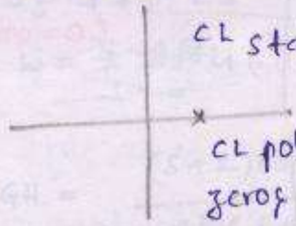


OL stable

OL pole = 0 ⇓ poles of CE = 0

$P = 0$

$N = -Z$



CL stable

CL poles = 0

zeros of CE = 0

$Z = 0$

$N = P$

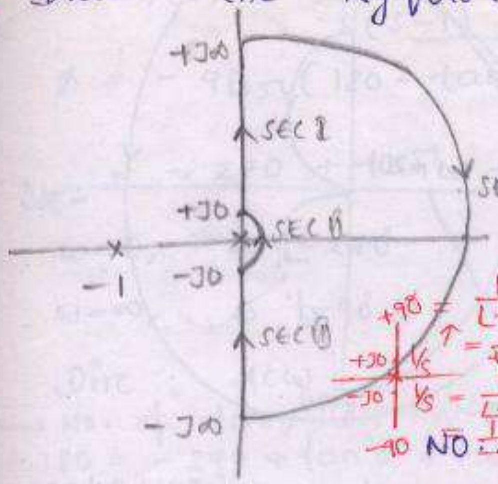
* To become a OL system stable, there should not be any OL pole in the RHS. The OL poles are nothing but poles of char. eq which must be zero on RHS. ie P must be '0' and $N = -Z$.

* To become a CL system stable, there should not be any CL pole in the RHS. The CL pole is nothing but a zero's of CE in the RHS. which must be '0' ie $Z = 0$, $N = P$.

Nyquist stability Criteria:-

It states that the no. of encirclements about the critical point $(-1+j0)$ in the $G(s)H(s)$ plane must be = to no. of p's of CE. [OL T/F p's which are in the RH - s- plane]. ie. $N = P$

Q. Draw the Nyquist plot for $G(s).H(s) = \frac{1}{s(s+1)}$



SEC-I

$\omega = 0+$ $\infty \angle -90$

$\omega = \infty$ $0 \angle -180^\circ$

E. Dire : cw

S.D → cw

SEC-III is mirror image of sec-I wrt real axis and the dire. is continuous.

SEC-II.

$+90 = \frac{1}{\angle +90}$, $\uparrow = \frac{1}{+90} = -90$

$+J0$ $1/0$, $-J0$ $1/0 = \frac{1}{\angle -0}$

$\omega = -0$, $\infty \angle +90$

$\omega = +0$, $\infty \angle -90$ Dire : cw

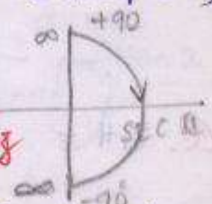
NO $= \frac{1}{\angle -90}$ = infinite 180° o^{le} s [half o^{le}]

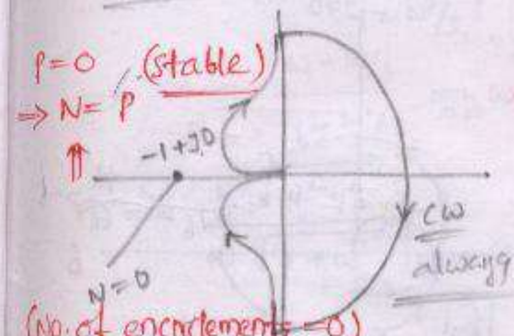
Neglect sec-IV = no. of poles at origin.

SEC-IV :

$\omega = +\infty$; neglect sec-IV b'coz

$\omega = -\infty$; magnitude is zero.





P=0 (Stable)

⇒ N = P

N=0

(No. of encirclements = 0)

The infinite half o^{le} should start at where mirror image is ending and it should be end at where actual polar plot is started.

* The dire. of infinite half o^{le} is always cw. irrespective of location of p's and z's.

$\frac{1}{s(s+1)}$ → P = 0 N = P, CL stable.

Q $GH = \frac{10}{s+5}$

$\omega = 0$, $2 \angle 0^\circ$

$\omega = \infty$, $0 \angle -90$

Dire : cw

Q. $GH = \frac{10}{(s+1)(s+2)}$

$\omega = 0$, $5 \angle 0^\circ$

$\omega = \infty$, $0 \angle -180$

Direc : cw

Q. $GH = \frac{10}{s^2(s+1)(s+2)}$ (2 half o'les)

$\omega = 0$, $\infty \angle -180$

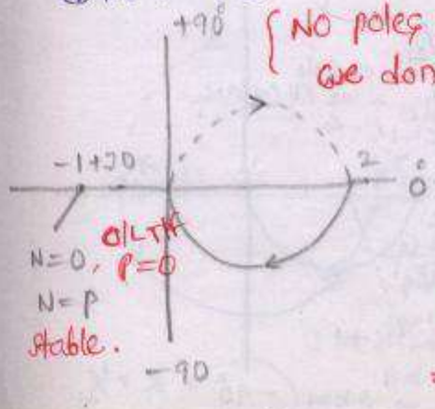
$\omega = \infty$, $0 \angle -360$

Dire : cw

N = P-Z ⇒ -2 = 0 - Z

⇒ Z = 2

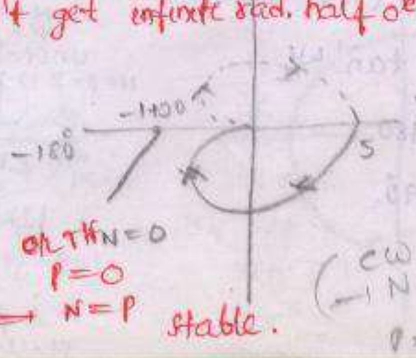
[No poles & zeros at origin so we don't get infinite rad. half o^{le}]



O/L TF N=0, P=0

N=P

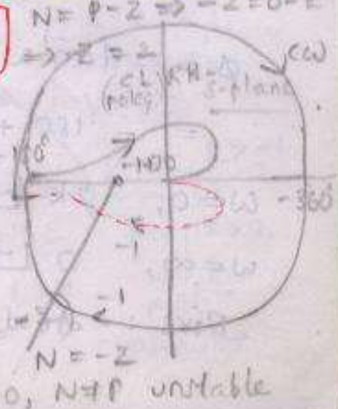
Stable.



O/L TF N=0

P=0

⇒ N=P Stable.



N = -2

P = 0, N ≠ P unstable

Q. $GH = \frac{1}{s^3(s+1)}$ 3 half O'les. $\rightarrow P = 0$

$\omega = 0, \quad \infty \angle -270^\circ$

$\omega = \infty, \quad 0 \angle -360^\circ$

Dire : CW

SD → CW

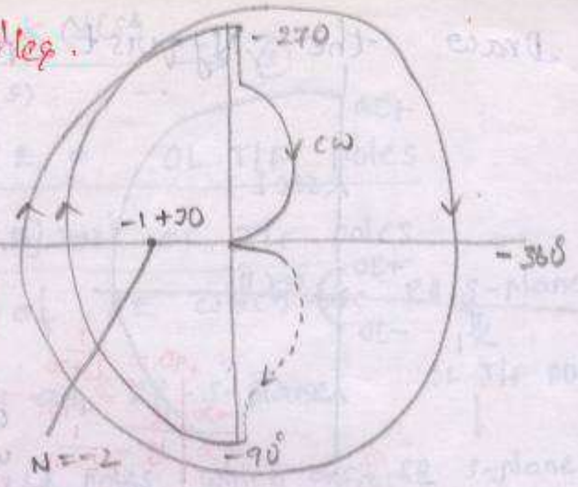
$P = 0$

$N \neq P$

unstable

$N = P - Z$

$-2 = 0 - Z \Rightarrow Z = 2$ | CL poles on RH - s-plane.



Q. $GH = \frac{k}{(s+1)(s+2)(s+3)}$

$\omega = 0, \quad \frac{k}{6} \angle 0^\circ$

$\omega = \infty, \quad 0 \angle -270^\circ$

Dire : CW

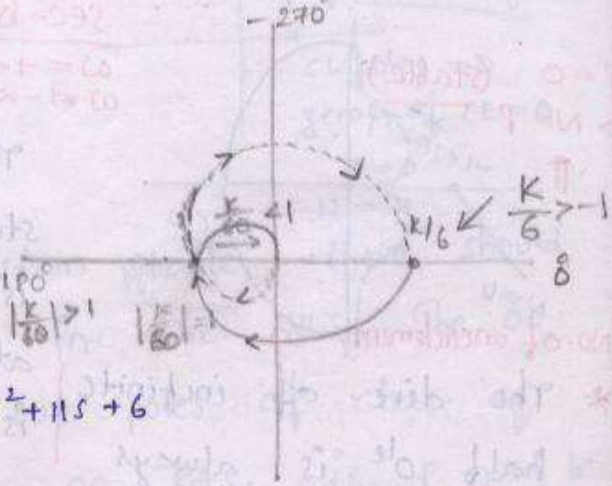
$(s+1)(s+2)(s+3) = s^3 + 6s^2 + 11s + 6$

$s^3 + 11s = 0$

$\omega_{pc} = \sqrt{11}$

$$M = \frac{k}{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}\Big|_{\omega=\sqrt{11}}} = 1$$

$= \frac{k}{60}$



For stable,

$N = 0$

$P = 0$

$N = P$ – stable

$\frac{k}{60} < 1 \Rightarrow k < 60$

and $\frac{k}{6} > -1 \Rightarrow k > -6$

$-6 < k < 60$

Q. find the range of k- value for $GH = \frac{k(s+2)}{(s+1)(s-1)} \rightarrow P = 1$ for System Stability.

$\phi = -\tan^{-1}\omega - (180 - \tan^{-1}\omega) + \tan^{-1}\omega/2$

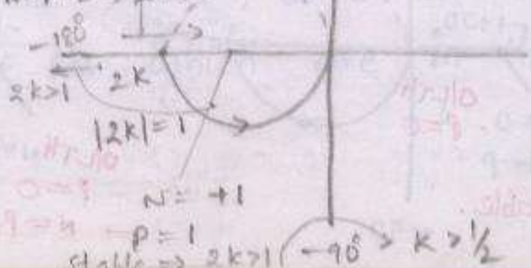
$= -(180 + \tan^{-1}\omega/2)$

$\omega = 0, \quad 2k \angle -180$

$\omega = \infty, \quad 0 \angle -90^\circ$

Dire : ACW

N = 0
P = 1
unstable
$N = P - Z \Rightarrow Z = 1$
1 CL pole at RH s-plane



N = +1

P = 1

stable ⇒ $2k > 1 \Rightarrow k > 1/2$

Q. $G(s)\cdot H(s) = \dfrac{k(s+3)}{s(s-1)} \longrightarrow P=1$

$\phi = -90 - (180 - \tan^{-1}\omega) + \tan^{-1}\omega/3$

$= -270 + \tan^{-1}\omega + \tan^{-1}\omega/3$

$\omega = 0, \quad \infty \angle -270^\circ$

$\omega = \infty, \quad 0 \angle -90^\circ$

Dire : ACW

→ No. of terms less than 2.

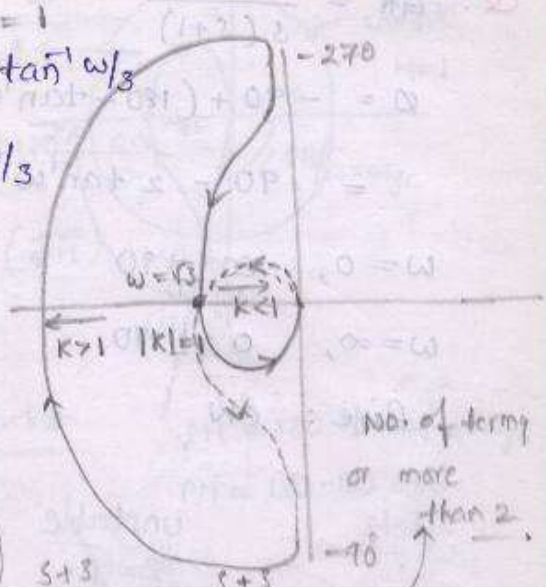
$-180 = -270 + \tan^{-1}\omega + \tan^{-1}\omega/3$

$90 = \tan^{-1}\left[\dfrac{\omega + \omega/3}{1-\omega^2/3}\right]$

$\Rightarrow \infty = \dfrac{\omega+\omega/3}{1-\omega^2/3} \Rightarrow \omega = \sqrt{3}$

$M = \dfrac{k\sqrt{\omega^2+9}}{\omega\sqrt{1+\omega^2}}\Big|_{\omega=\sqrt{3}}$

$M = k$



$\dfrac{s+3}{s(s-1)} = \dfrac{s+3}{-s(1-s)}$

$= +s(s+3)(1+s) = 0$

$(s^3 + 4s^2 + 3s$

$s \to j\omega, \quad -j\omega^3 + 3j\omega = 0$

$\Rightarrow \omega = \sqrt{3}$

for $k>1$,

$N = +1 \quad (+2-1)$

$P = 1$

stable

for $k<1$

$N = -1$

$P = 1$

$N = P - Z \Rightarrow -1 = 1 - Z \Rightarrow Z = 2$

2 CL p' on RH s-plane

↑ unstable.

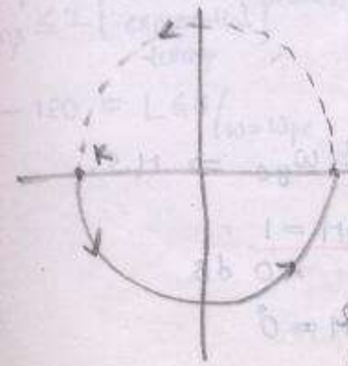
Q. $GH = \dfrac{k(s+5)}{s-5} \longrightarrow P=1$

$\phi = -180 + 2\tan^{-1}\omega/5$

$\omega = 0, \quad k \angle -180$

$\omega = \infty, \quad k \angle 0^\circ$

Direction : ACW



$N = 1$

$P = 1$

stable $k>1$

$1 < k \le \infty$

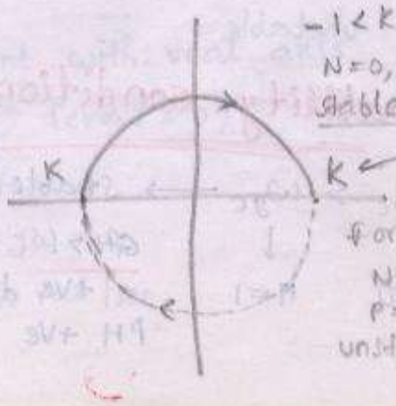
Q. $GH = \dfrac{k(s-2)}{s+2} \longrightarrow P=0$

$\phi = +180 - 2\tan^{-1}\omega/2$

$\omega = 0, \quad k \angle 180^\circ$

$\omega = \infty, \quad k \angle 0^\circ$

Direction : CW



$-1 < k < 1$,

$N=0, P=0$

stable

$k \leftarrow k > -1$

for, $k>0$,

$N \neq P$

$P=0$

unstable

Q. $GH = \dfrac{s-1}{s(s+1)} \longrightarrow P=0$

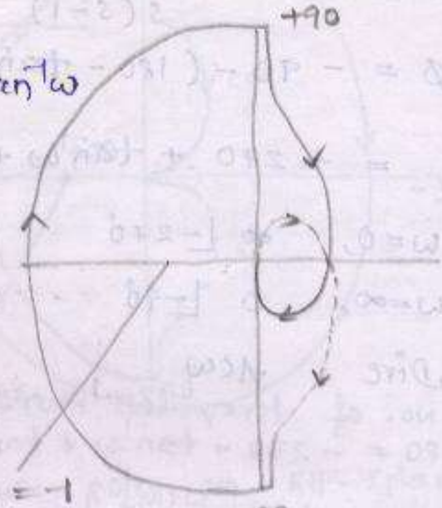
$\phi = -90 + (180 - \tan^{-1}\omega) - \tan^{-1}\omega$

$= 90 - 2\tan^{-1}\omega$

$\omega = 0, \quad \infty \angle +90$

$\omega = \infty, \quad 0 \angle -90$

Dire : CW



unstable $\leftarrow$

Z = 1

Q. $GH = \dfrac{1}{s(s-1)} \longrightarrow P=1$

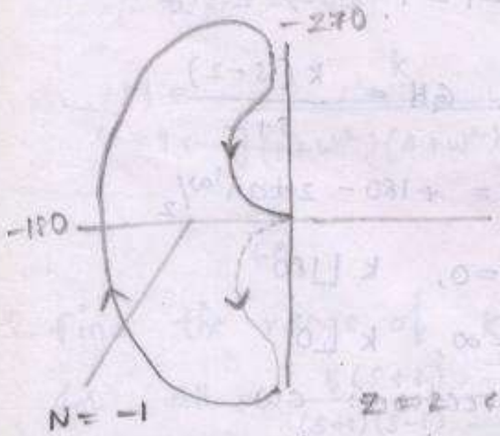
$\phi = -90 - (180 - \tan^{-1}\omega)$

$= -270 + \tan^{-1}\omega$

$\omega = 0, \quad \infty \angle -270$

$\omega = \infty, \quad 0 \angle -180$

Dire : ACW



N = -1 Z = 2 CL RH

P = 1 unstable

Q. $GH = \dfrac{1}{s(-s+1)} \longrightarrow P=1$

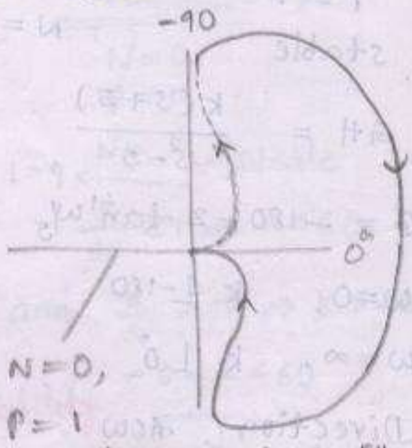
$\phi = -90 - (-\tan^{-1}\omega)$

$= -90 + \tan^{-1}\omega$

$\omega = 0, \quad \infty \angle -90$

$\omega = \infty, \quad 0 \angle 0$

Dire : ACW



N = 0,

P = 1

unstable, Z = 1 CL RH.

$\Rightarrow$ stability conditions :

$\omega_{pc} > \omega_{gc} \longrightarrow$ stable

$\omega_{pc} \downarrow -180$, $\omega_{gc} \downarrow M=1$

GH > 1

+ve dB

PM +ve

$\omega_{pc} = \omega_{gc} \Rightarrow$ M.S.

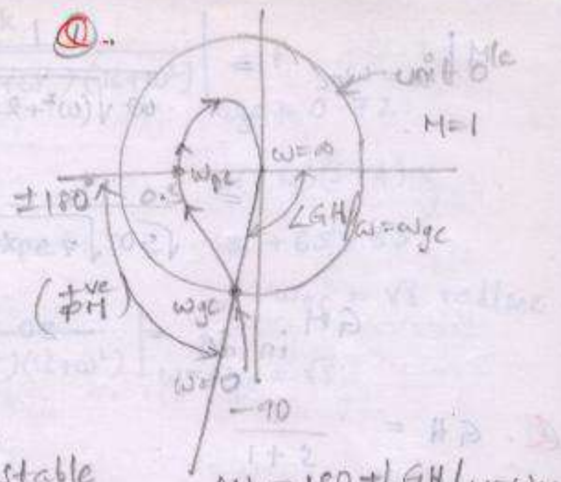
GH = 1

= 0 dB

PM = 0°

$\omega_{pc} < \omega_{gc} \Rightarrow$ unstable

Q.



GM < 1

-ve dB

PM -ve

$\omega_{pc} > \omega_{gc}$

stable

$GM = \dfrac{1}{M|_{\omega=\omega_{pc}}}$

$= \dfrac{1}{<1|_{\omega=\omega_{pc}}} > 1$ stable

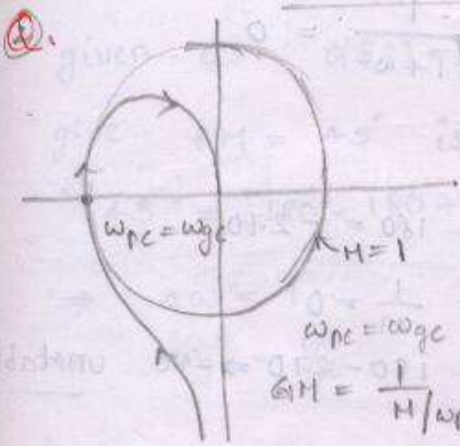
$= \dfrac{1}{0.5} = 2$

$PM = 180 + \angle GH|_{\omega=\omega_{gc}}$

$PM = 180 - 100 = 80^\circ$

stable

Q.



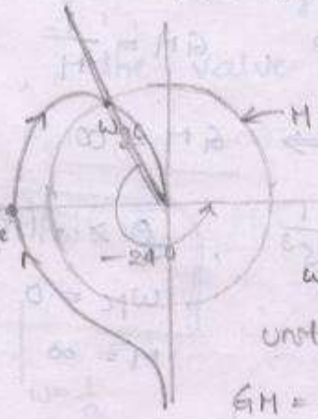
$\omega_{pc} = \omega_{gc}$

$GM = \dfrac{1}{M|_{\omega_{pc}}} = \dfrac{1}{1} = 1$

$PM = 180 - 180 = 0^\circ$

M.S.

Q.



$PM = 180 - 240$

$= -60$

-ve

$\omega_{pc} < \omega_{gc}$

unstable.

$GM = \dfrac{1}{M|_{\omega=\omega_{pc}}}$

$= \dfrac{1}{10} = 0.1 < 1$

whenever plot intersect $\pm 180^\circ$, with a mag. of <1 (GM>1) then the system is stable, if $M=1$, then M.S, if mag. $(M) > 1$ (GM<1) unstable.

Q. find the gain margin for $GH = \dfrac{1}{s(s+5)(s+10)}$

ω_{pc}

no. of terms ≤ 2 [T/F consists exponential terms] ≥ 2 Intersection point with real axis (imaginary terms = 0)

$-180 = \angle GH|_{\omega=\omega_{pc}}$

$s^3 + 15s^2 + 50s$

$-j\omega^3 + 50j\omega = 0$

$\omega_{pc} = \sqrt{50}$ rad/sec

$$M\Big|_{\omega=\omega_{pc}} = \frac{1}{\omega\sqrt{(\omega^2+25)(100+\omega^2)}}$$

$$= \frac{1}{\sqrt{50}\sqrt{75\times150}} \Rightarrow GM = \frac{1}{M\big|_{\omega=\omega_{pc}}} = 750$$

$$GM_{\text{in dB}} = -20\log\frac{1}{\sqrt{50\times75\times150}} = 20\log 750\ (+ve)$$

Q. $GH = \frac{1}{s+1}$

$-180 = -\tan^{-1}\omega$

$\omega_{pc} = \infty$, $GM = \frac{1}{M}$, $M = \frac{1}{\sqrt{1+\omega^2}} = 0$

$\Longrightarrow GM = \infty$

Q. $GH = \frac{1}{s^3}$, $\boxed{\begin{array}{l} 0 > -180 \\ \omega_{pc} = 0 \end{array}}$, $-180 = -270$

$M = \infty$, $GM = \frac{1}{M} = 0$

$PM = 180 - 270 \Rightarrow -ve$ unstable

Q. $GH = \frac{1}{s(s+1)}$

$\omega_{gc} \longrightarrow$ using magnitude condition

$|GH|_{\omega=\omega_{gc}} = 1$ $\qquad \left|\frac{1}{\omega\sqrt{1+\omega^2}}\right|_{\omega=\omega_{gc}} = 1$

$PM = 180 + \angle GH\big|_{\omega=\omega_{gc}}$ $\Rightarrow \omega = 0.78$ rad/sec.

$PM = 180 - 90 - \tan^{-1}\omega\big|_{\omega=\omega_{gc}=0.78}$

$= 52^\circ$.

Q. The OL T/F of a system is $GH = \frac{k}{s(s+2)(s+4)}$.

Determine the value of k, (i) so that $PM = 60^\circ$,

(ii). so that $GM = +20$ dB

$PM = 60 = 180 - 90 - \tan^{-1}\omega/_2 - \tan^{-1}\omega/_4$

$\Rightarrow 30 = \tan^{-1}\left[\frac{\omega/_2 + \omega/_4}{1 - \omega^2/_8}\right]$

$\Rightarrow \omega = \omega_{gc} = 0.72$ rad/sec

Magnitude condi. $\left| \dfrac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega_{gc}=0.72} = 1$

$\Rightarrow k = 6.2$

$s(s+2)(s+4)$

s^3+6s^2+8s

$GM = -20 \log M/_{\omega=\omega_{pc}}$

$\Rightarrow \omega_{pc} = \sqrt{8}$ rad/sec

$20 = -20 \log \left. \dfrac{k}{\omega\sqrt{(4+\omega^2)(16+\omega^2)}} \right|_{\omega=\omega_{pc}=\sqrt{8}}$

$\Rightarrow k = 4.8.$

Q. The OL T/F of a unity f/b control system is given as $G(s) = \dfrac{as+1}{s^2}$, the value of 'a' to give PM = 45° is –

$45 = 180 - 180 + \tan^{-1}(a\omega)/_{\omega=\omega_{gc}}$

$\Rightarrow a\omega = 1$

$\omega_{gc} = 1/a$

$\left. \dfrac{\sqrt{(a\omega)^2+1}}{\omega^2} \right|_{\omega=\frac{1}{a}} = 1$

$\Rightarrow a^2 = \dfrac{1}{\sqrt{2}}$

Q. In the GH plane, the Nyquist plot of T/F $GH = \dfrac{\pi e^{-0.25s}}{s}$ passes through the –ve real axis, at a point – ?

1). $(-0.25, j0)$

2). $(-0.5, j0)$

3). $(-1, j0)$

4). $(-2, j0)$

$-180 = -90 - 0.25\omega \times \dfrac{180}{\pi} \Big|_{\omega=\omega_{pc}}$

$\Rightarrow \omega_{pc} = 2\pi$ rad/sec

exp, m=1

$M = \dfrac{\pi}{\omega}\Big|_{\omega_{pc}=2\pi} = 0.5 \quad (-0.5, j0)$

$e^{j\theta} = \cos\theta + j\sin\theta$

$\sqrt{\cos^2\theta+\sin^2\theta}$

exponential term never effect magnitude but effects phase angle

* whenever the T/F not gives the mag. of '1' at any freq. then consider $\omega_{gc} = 0$.

15 - 06 - 07.

State Space Analysis:

State gives the future behaviour of the system based on past history and present i/p of the system. * The initial (past) state of system is described by state variable.

~~Limitations~~

→ No. of state variables:-

If electrical n/w given, the no. of state variables = sum of the inductors & conductors

if a differential eq. given, the no. of state var.s = order of the differential eq.

Limitations of T/F Analysis:-

(1). The T/F analysis is valid only for LTI systems, where as SSA is valid for dynamic [linear, non-linear, time varient, time invarient] systems.

(2). The T/F analysis cannot give any idea about controllability and observability.

(3). T/F Analysis is more suitable for SISO systems. where as SSA suitable for MIMO.

Standard form of D. state model:-

state eq (or) dynamic eq

$$\underset{\substack{\downarrow \\ \text{Differential} \\ \text{state vector}}}{\dot{X}} = \underset{\substack{\downarrow \\ \text{state} \\ \text{Matrix}}}{A}X + \underset{\substack{\downarrow \\ \text{i/p} \\ \text{matric}}}{B}U$$

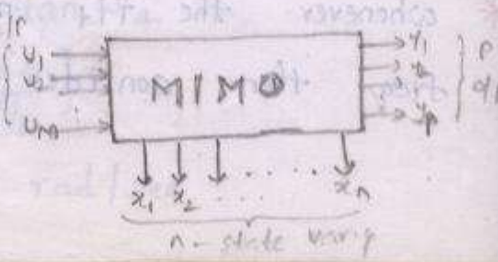
(o/p eq.)

$$\underset{\substack{\downarrow \\ \text{o/p} \\ \text{vector}}}{Y} = \underset{\substack{\downarrow \\ \text{o/p} \\ \text{Matrix}}}{C}\overset{\substack{\text{state vector} \\ \uparrow}}{X} + \underset{\substack{\downarrow \\ \text{Transmission Matrix}}}{D}\overset{\substack{\text{i/p vector} \\ \uparrow}}{U}$$

NOTE: D is always zero, if the circuit not present the active elements.

Order of Matrices:-

Consider the MIMO system,



state vector = $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$ o/p vector = $\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}_{p\times 1}$ i/p vector = $\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}_{m\times 1}$

$$* \quad \underset{n\times 1}{\dot{X}} = \overset{n\times n}{A}\underset{n\times 1}{X} + \overset{n\times m}{B}\underset{m\times 1}{U} \qquad \underset{p\times 1}{Y} = \overset{p\times n}{C}\underset{n\times 1}{X} + \overset{p\times m}{D}\underset{m\times 1}{U}$$

Q. find the order of Matrices:-

(1). $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 5y = 10\,u(t)$

$n = 2$. i/p $= 1$ (m), o/p $= 1$ (p)

$$\underset{2\times 1}{\dot{X}} = \overset{2\times 1}{\underset{2\times 2}{A}}\, X + \overset{1\times 1}{\underset{2\times 1}{B}}\, U \;; \quad \underset{1\times 1}{Y} = \overset{2\times 1}{\underset{1\times 2}{C}}\, X + \overset{1\times 1}{\underset{1\times 1}{D}}\, U$$

Q. Obtain the state model by using

$y''' + 2y'' + 3y' + y = u$

Let $n = 3$. (no. of state var.s = no. of differential state variables)

$x_1 = y$, $\dot{x}_1 = \dot{y} = x_2$

$\dot{x}_2 = \ddot{y} = x_3$

$\dot{x}_3 = \dddot{y}$ To get $\dot{x}_3$ relationship with state var.s sub. all 4 eq.s in the given system.

$\Rightarrow \dot{x}_3 + 2\dot{x}_2 + 3\dot{x}_1 + x_1 \Rightarrow \dot{x}_3 + 2x_3 + 3x_2 + x_1 = u$

$\Rightarrow \dot{x}_3 = u - x_1 - 3x_2 - 2x_3$

$\dot{X} = AX + BU$ $(n = 3,\ p = 1,\ m = 1)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad \& \quad [y] = [1\ 0\ 0]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q. $y'''' + 10y''' - 6y'' + 7y' + 5y = 10\,u(t)$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & -7 & +6 & -10 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \text{ and } C = [1\ 0\ 0\ 0]$$

Q. Obtain the state model for given T/F,

$$\frac{y(s)}{u(s)} = \frac{10s+5}{s^3+6s^2+7s+8}$$

$\dot{x}_3 \quad \dot{x}_2 = x_3 \quad \dot{x}_1 = x_2$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \qquad y^n = s^n = \dot{x}_n$$

$$y(s) = 10x_2 + 5x_1$$

$$u(s) = \dot{x}_3 + 6x_3 + 7x_2 + 8x_1 \Rightarrow \dot{x}_3 = u(s) - 8x_1 - 7x_2 - 6x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -7 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u] \quad \& \quad [y] = [5 \quad 10 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q. $T/F = \dfrac{s^2+5s+10}{s^4+3s^3+6s^2+5}$

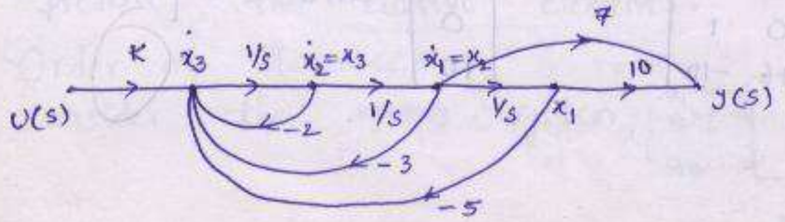
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & 0 & 6 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = [10 \quad 5 \quad 1 \quad 0]$$

Q. $T/F = \dfrac{7s+6}{(s+1)(s+2)(s+3)}$ directly $\rightarrow$ $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$= \frac{7s+6}{s^3+6s^2+12s+6} \qquad C = [6 \quad 7 \quad 0]$$

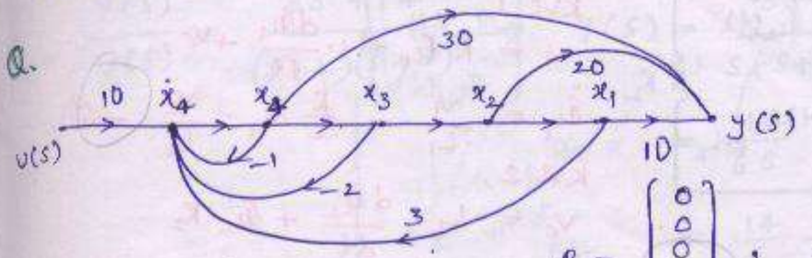
Q. $T/F = \dfrac{7s+6}{(s+2)^3\,(s+5)}$ $\quad A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix}$

Q. Obtain the A, B, C matrices for given signal flow graph.



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -5 & -3 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ k\end{bmatrix}[u] \; ; [y] = [10 \;\; 7 \;\; 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Q.



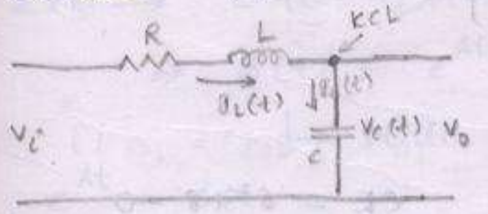
$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 3 & 0 & -2 & -1\end{bmatrix}$$

$$B = \begin{bmatrix}0\\ 0\\ 0\\ 10\end{bmatrix} ; \quad C = [10 \;\; 20 \;\; 0 \;\; 30]$$

Procedure for obtain the state eq for electrical n/w :-

1. Select the state var.s as volt. across capacitor and current through inductor. The no. of state var.s = sum of inductors and capacitors.
2. write the independent KCL & KVL, Apply KCL at capacitor junction and KVL through inductor
3. The resultant eq. must consists state var.s differential state var.s, i/p and o/p var.s

⇒ Obtain the state model for the given electrical n/w.



KCL at J_c

$$i_L(t) = i_c(t) = C \cdot \frac{dV_c(t)}{dt}$$

$$\dot{V}_c(t) = \frac{i_L(t)}{C} \rightarrow ①$$

KVL through inductor

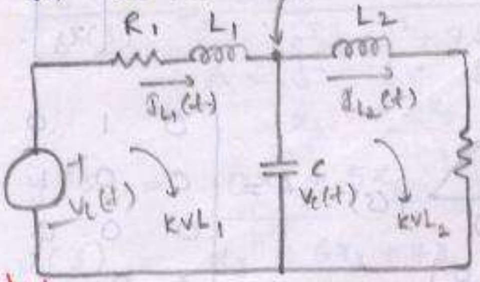
$$V_i(t) - R\, i_L(t) - L\frac{d i_L(t)}{dt} - V_c(t) = 0$$

$$\dot{i}_L(t) = \frac{V_i(t)}{L} - \frac{R}{L} I_L(t) - \frac{V_c(t)}{L} \rightarrow ②$$

$$\begin{bmatrix}\dot{V}_c(t)\\ \dot{i}_L(t)\end{bmatrix} = \begin{bmatrix}0 & 1/C\\ -1/L & -R/L\end{bmatrix}\begin{bmatrix}V_c(t)\\ i_L(t)\end{bmatrix} + \begin{bmatrix}0\\ 1/L\end{bmatrix}[V_i(t)]$$

$$[V_o(t)] = [1 \;\; 0]\begin{bmatrix}V_c(t)\\ i_L(t)\end{bmatrix}$$

whenever same kind of elements connected in series or parallel then it should be treated as single element.



KCL,

$i_{L_1} = i_{L_2} + i_c$

$i_{L_1} = i_{L_2} + C \cdot \frac{dv_c}{dt}$

$\dot{v}_c = \frac{i_{L_1}}{C} - \frac{i_{L_2}}{C}$ —①

KVL 1,

$v_1 = R\, i_{L_1} + L_1 \cdot \frac{di_{L_1}}{dt} + v_c$

$\dot{i}_{L_1} = \frac{v_1}{L_1} - \frac{R}{L} i_{L_1} - \frac{v_c}{L_1}$ —②

KVL 2,

$v_c = L_2 \cdot \frac{di_{L_2}}{dt} + i_{L_2} \cdot R_2$

$\dot{i}_{L_2} = \frac{v_c}{L_2} - \frac{R_2}{L_2} \cdot i_{L_2}$ —③.

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_{L_1} \\ \dot{i}_{L_2} \end{bmatrix} = \begin{bmatrix} 0 & 1/C & -1/C \\ -1/L_1 & -R/L_1 & 0 \\ 1/L_2 & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} v_c \\ i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_1 \\ 0 \end{bmatrix} v_1$$

T/f from State Model :-

$$\frac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D$$

$$\frac{Y(s)}{U(s)} = C \cdot \frac{Adj[sI-A]}{|sI-A|} B + D$$

$|sI - A| = 0 \longrightarrow CE \xrightarrow{\text{Roots of}}$

$\xrightarrow{CE}$ CL poles $\rightarrow$ eigen values.

Q. Consider the state model that is,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} +2 & +3 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} [u] \; ; \; [y] = [1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i). find the nature of system (ii). Obtain stability (iii). Obtain the T/f.

$$T/f = [1 \quad 1] \frac{\begin{bmatrix} s-2 & -3 \\ 4 & s+2 \end{bmatrix}}{s^2+8} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\frac{\begin{bmatrix} 3s - 6 - 15 \\ 12 - 5s + 10 \end{bmatrix}}{s^2+8} = \frac{8s+1}{s^2+8}$$

$CE = s^2 + 8 = 0$

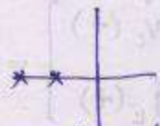
$\Rightarrow s = \pm j\sqrt{8}$

$\xi = 0$ undamped $\rightarrow$ M.S.

Q. Obtain the T/f,

$$[\dot{x}] = \begin{bmatrix} 0 & 3 \\ -2 & -5 \end{bmatrix} [x] + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u] \; ; \quad y = [2 \quad 1][x]$$

$$T/f = [2 \quad 1] \frac{\begin{bmatrix} s+5 & +3 \\ -2 & s \end{bmatrix}}{s^2+5s+6} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{3s+14}{(s+2)(s+3)}$$



over damped $\xi > 1$

stable

Q. find the unit step response for the above state model and also draw the R.L diagram.

$$\frac{Y(s)}{U(s)} = \frac{3s+14}{(s+2)(s+3)} \Rightarrow Y(s) = \frac{3s+14}{s(s+2)(s+3)}$$

$$= \frac{14}{6s} - \frac{4}{s+2} + \frac{5}{3}\cdot\frac{1}{(s+3)}$$

$$= \frac{14}{6} - 4e^{-2t} + \frac{5}{3}e^{-3t}$$

Here there is NO 'K' value so, NO R.L.

In the above system, there is NO system gain parameter, hence RL diagram is nothing but loc. of p & z's.

Solution to the state eq :-

$\dot{x} = Ax + Bu \leftarrow$ Non homogeneous state eq.

(1). L.T. :-

$$x(t) = \boxed{L^{-1}\left[[sI-A]^{-1}x(0)\right]} + \boxed{L^{-1}\left[[sI-A]^{-1}BU(s)\right]}$$

ZIR → due to initial condi. ; ZSR → due to i/p

(2). Classical Method :-

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}\,.\,BU(\tau).d\tau$$

ZIR : Natural (or) free force, system impulse.

ZSR : forced response,

$$ZIR \longrightarrow L^{-1}\left[[sI-A]^{-1}x(0)\right] = e^{At}x[0]$$

$$\Rightarrow \phi(t) = e^{At} = L^{-1}\left[[sI-A]^{-1}\right]$$

↳ state transition Matrix.

$$e^{At} = \phi(t)$$

$$e^{A(t-\tau)} = \phi(t-\tau)$$

$$L^{-1}[sI-A]^{-1} = \phi(t) \Rightarrow [sI-A]^{-1} = \phi(s)$$

$$ZSR \rightarrow L^{-1}\left[\phi(s).BU(s)\right] = \int_0^t \phi(t-\tau).BU(\tau).d\tau$$

Properties of S.T.M :-

$$STM\ \ \phi(t) = e^{At} = L^{-1}\left[[sI-A]^{-1}\right]$$

1. $\phi(0) = I$ [Identity Matrix]
2. $\phi^k(t) = (e^{At})^k = e^{A(kt)} = \phi(kt)$.

3. $\phi^{-1}(t) = \phi(-t)$

4. $\phi(t_1+t_2) = \phi(t_1).\phi(t_2)$.

5. $\phi(t_2-t_1).\phi(t_1-t_0) = \phi(t_2-t_0)$

Q. Obtain the time response for the given system,

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x$ where $x[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$y = [1 \ -1]\, x$

$x(t) = e^{At} x(0) + L^{-1}[\phi(s).B\,U(s)]$ ← soln. to Non. homo. eq

$\dot{x} = Ax + BU \rightarrow$ Non- homogeneous . eq.

$\dot{x} = Ax \rightarrow$ Homogeneous eq . , $U = 0$.

Soln. of homogeneous eq : $x(t) = e^{At} x(0)$.

The given system is homogeneous be cause $U(s)=0$,

STM $\phi(t) = e^{At} = L^{-1}[[sI-A]^{-1}]$ $\quad x(t) = e^{At} x(0)$

$$= L^{-1}\begin{bmatrix} \frac{s}{s^2+2} & \frac{+1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix} = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

{ The correct STM is, which gives Identity Matrix for $t=0$ }

$\Rightarrow x(t) = e^{At}.x(0) =$

$y(t) = [1 \ -1]\, x(t) = \frac{3}{\sqrt{2}} \sin\sqrt{2}t$.

Q. find the time response for given

$\dot{x} = \begin{bmatrix} 0 & -1 \\ +2 & 5+3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u$, $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $y(t) = [0 \ 1]\, x(t)$

$x(t) = e^{At}.x(0) + L^{-1}[\phi(s).B\,U(s)]$

$$\phi(t) = e^{At} = L^{-1}\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{+1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

→ If we required to find STM, substitute $t=0$ in the given options. $\phi(t)$ at $t=0$, must be the identity matrix.

$$x(t) = ZIR = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$ZSR = L^{-1}\begin{bmatrix} \frac{5}{s(s+1)(s+2)} \\ \frac{5}{(s+1)(s+2)} \end{bmatrix} = L^{-1}\begin{bmatrix} \frac{2.5}{s} - 5\frac{1}{s+1} + \frac{2.5}{s+2} \\ \frac{5}{s+1} - \frac{5}{s+2} \end{bmatrix} = \begin{bmatrix} 2.5 - 5e^{-t} + 2.5e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}$$

$$x(t) = ZIR + ZSR$$

$$= \begin{bmatrix} 2.5 - 3e^{-t} + 1.5e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix} \quad \& \quad Y(t) = 3e^{-t} - 3e^{-2t}$$

$y(t) = [0\ 1]\ x(t)$

Controllability:-

A system is said to be controllable if it is possible to transfer the initial state to desired state in a finite time interval by the controlled i/p.

Kalman's test for controllability:-

$$Q_c = [B \quad AB \quad A^2B \ \cdots \ A^{n-1}B]$$

Rank of Q_c = Rank of A

$|Q_c| \neq 0 \rightarrow$ controllable.

Q. Check controllability; $\quad T/f = \dfrac{1}{s^3 + 2s^2 + 3s + 4}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$n = 3$.

$(B\ \ AB,\ A^2B)$

$$Q_c = \begin{array}{c} \begin{matrix} B & AB & A^2B \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix} \end{array}; \quad |Q_c| \neq 0 \rightarrow \text{controllable}$$

Q. $$\dot{x} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Q_c = \begin{array}{c} \begin{matrix} B & AB \end{matrix} \\ \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \end{array} \rightarrow \text{controllable}$$

Q. $\dot{x}_1 = -2x_1 + U$, $\dot{x}_2 = 3x_1 - 5x_2$

$$A = \begin{bmatrix} -2 & 0 \\ 3 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} : \quad Q_c = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \rightarrow \text{controllable}$$

Observability :-

A system is said to be observable, if it is possible to determine initial states of the system by observing the o/p's in a finite time interval.

Kalman's Test for Observability :-

$$Q_o = [\, C^T \;\; A^T C^T \;\; (A^T)^2 C^T \; \ldots \; (A^T)^{n-1} C^T \,]$$

(or)

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Rank of Q_o = Rank of A

$|Q_o| \neq 0$

$\rightarrow$ Observable

Q. check the controllability & observability,

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} U \; : \; Y = [1 \;\; 1]\, x$$

$$Q_c = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}, \quad |Q_c| = 0 \longrightarrow \text{Not controllable}$$

$$Q_o = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad |Q_o| = 0 \longrightarrow \text{Not observable.}$$

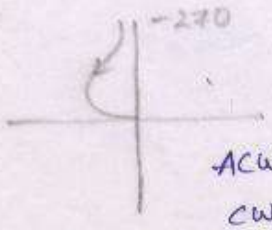
Q. $\dot{x}_1 = -2x_1 + x_2 + u$

$\dot{x}_2 = -x_2 + u$

$y = x_1 + x_2$

$$Q_c = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \longrightarrow |Q_c| = 0 \;\; \text{Not controllable}$$

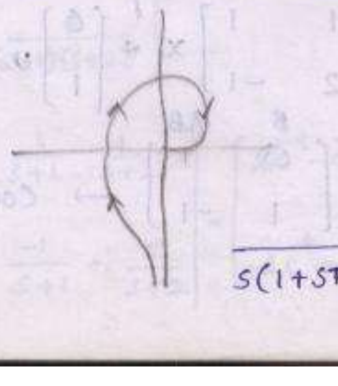
$$Q_o = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \rightarrow |Q_o| \neq 0 \;\; \text{observable.}$$



ACW → zero

CW → pole

$$\frac{K(1+sT_1)}{s^3}$$



$$\frac{K}{s(1+sT_1)(1+sT_2)(1+sT_3)}$$

Compensator :-

1. Lead → high pass → +ve angle $\xrightarrow{\text{given by}}$ zero
2. Lag → Low pass → -ve angle $\xrightarrow{\text{given by}}$ pole
3. Lead-Lag → B.P. → $T_{Lead} > T_{Lag}$
4. Lag-Lead → low pass, B.S, High pass → $T_{Lag} > T_{Lead}$ (lead, lag)

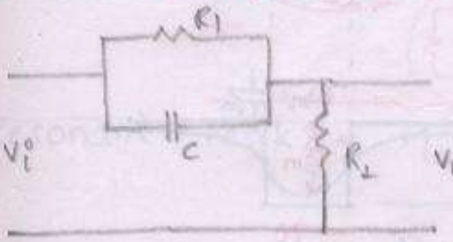
Each compensator gives the one finite pole and one finite zero.

When a sinusoidal i/p is applied to the n/w it produces a sinu. steady state o/p having a ph. lead w.r.t. i/p then the n/w is called lead compensator. The lead compensator speedup the transient response and increase the margin of system stability and also increases the error const. [i.e. ss error decreases].

If the ss o/p has the ph. lag then the n/w is called lag compensator. The lag compensator improves the ss behaviour without effecting the transient response.

The lag-lead or Lead-lag (both ph. lag & lead occurs but in different freq. regions) improves the both transient and ss behaviour.

Lead Compensator :-



S_1: T/F

S_2: T-const form

S_3: locate P/Z - s-plane

S_4: B.P & Identify filter.

S_5: $\omega_m \rightarrow \phi|_{\omega = \omega_m}$

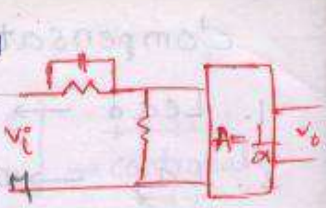
$\rightarrow M|_{\omega = \omega_m}$

$$\frac{V_0(s)}{V_i(s)} = \frac{R_2}{R_2 + \frac{R_1}{sCR_1 + 1}}$$

$$= \frac{R_2(1+sCR_1)}{R_1 + R_2 + sCR_1R_2} = \frac{R_2(1+sCR_1)}{(R_1+R_2)\left(1+\frac{R_2}{R_1+R_2}\cdot sCR_1\right)}$$

Let α - Lead const. $= \frac{R_2}{R_1+R_2} < 1$

τ - lead time const. $= R_1 C$



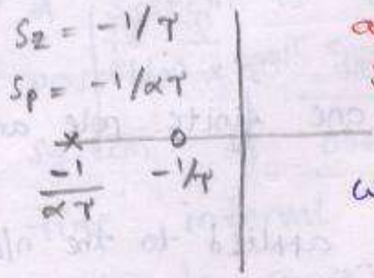
$$\frac{V_0(s)}{V_i(s)} = \frac{(\alpha)(1+\tau s)}{(1+\alpha\tau s)} \cdot \left(\frac{1}{\alpha}\right)$$

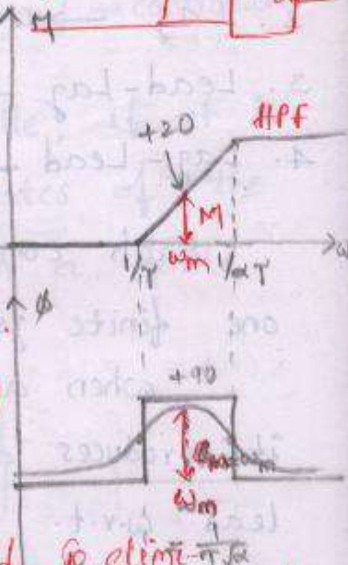
amplifier gain

high pass filter

$S_z = -1/\tau$

$S_p = -1/\alpha\tau$





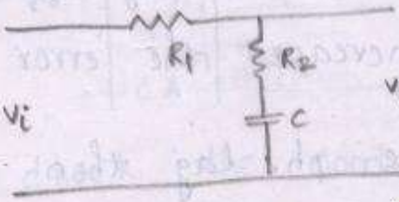
$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$; $M = 10\log 1/\alpha$.

$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$

α - Attenuation factor

b'coz α is <1. The main dis.adv in lead comp. is signal strength is attenuated. To eliminate attenuation we required to connect amplifier with gain of $1/\alpha$ in series to compen.

Lag compensator:-



$$\frac{V_0(s)}{V_i(s)} = \frac{R_2 + 1/sC}{R_1+R_2+1/sC}$$

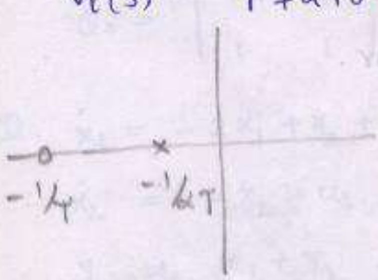
$$= \frac{1+sCR_2}{1+\frac{R_1+R_2}{R_2}\cdot sCR_2}$$

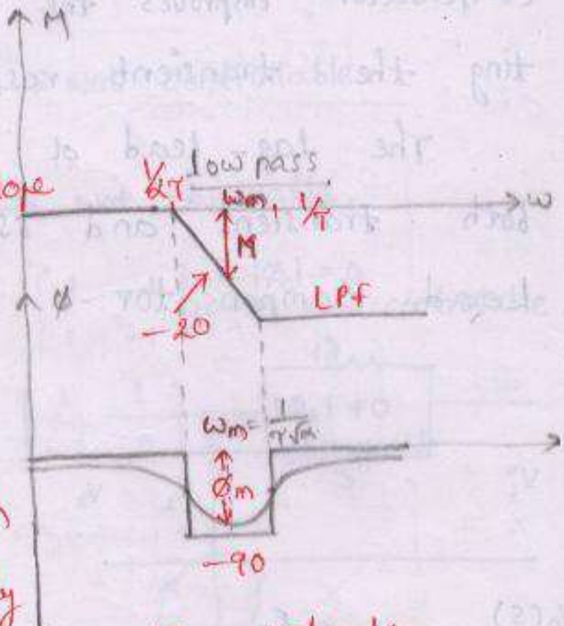
α - lag constant $= \frac{R_1+R_2}{R_2} > 1$

τ - lag time constant $= R_2 C$

$$\frac{V_0(s)}{V_i(s)} = \frac{1+\tau s}{1+\alpha\tau s}$$

Initial slope $= 0$





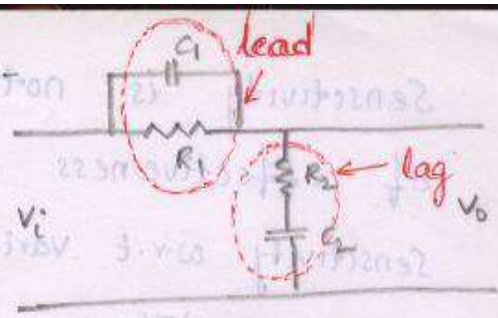
In compensators zero location is fixed, the change is only in poles location.

$M = 10\log 1/\alpha$

$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$

$\phi_m = \sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)$.

Lead-Lag compensator:-

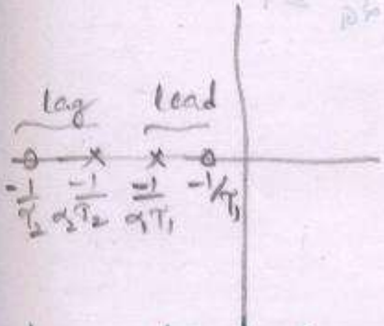


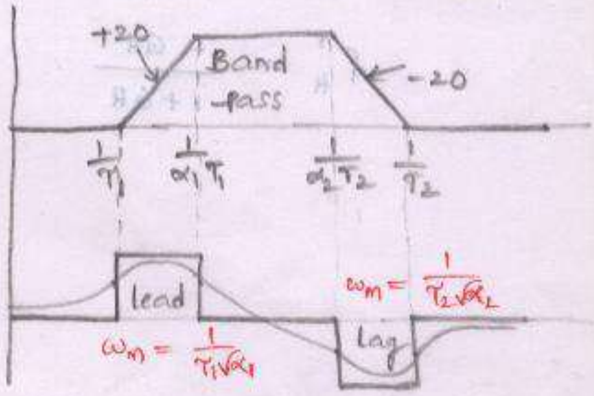
$\tau_{lead} > \tau_{lag}$

$$\frac{V_0(s)}{V_i(s)} = \underbrace{\frac{1+\tau_1 s}{1+\alpha\tau_1 s}}_{lead} \cdot \underbrace{\frac{1+\tau_2 s}{1+\alpha\tau_2 s}}_{lag}$$

τ_1 – lead $\tau = R_1C_1$ $\quad\quad$ τ_2 – lag $\tau = R_2C_2$

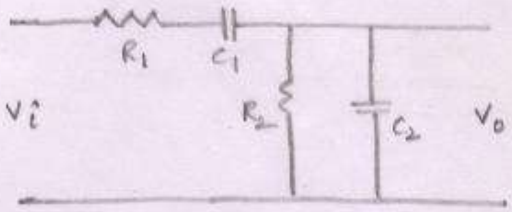
α_1 – lead const. $= \dfrac{R_2}{R_1+R_2} < 1$, α_2 – lag const. $= \dfrac{R_1+R_2}{R_2} > 1$

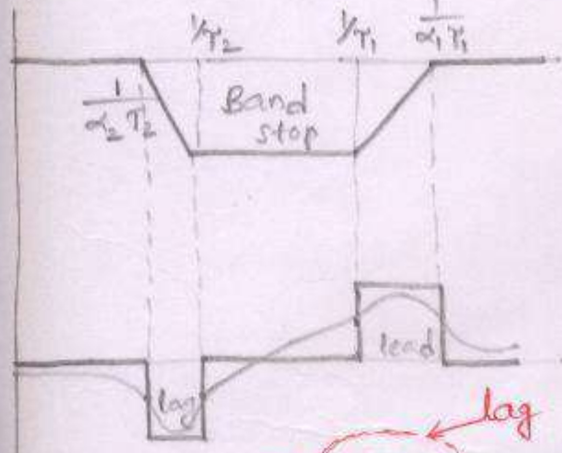


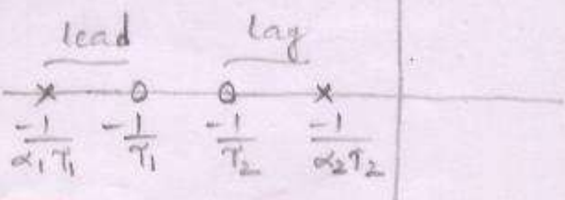


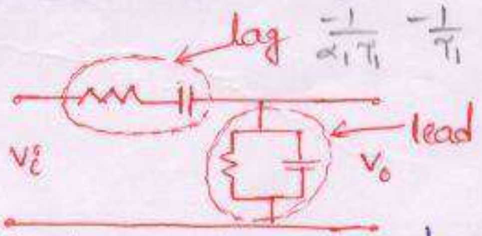
Lag-lead compensator:-

$\tau_{lag} > \tau_{lead}$









Resonant Peak $= M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$B.W. = \omega_b = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^2}}$$

$\uparrow B.W \propto \dfrac{1}{t_r \downarrow}$

Smallest $\xi \Rightarrow BW\uparrow$

Sensitivity is nothing but a measurement of effectiveness of f/b.

Sensitivity w.r.t variations in $G(s) = S_G^T = \frac{\partial T/T}{\partial G/G}$

$$= \frac{\partial T}{\partial G} \times \frac{G}{T}$$

$$S_H^T = \frac{\partial T/T}{\partial H/H} = \frac{\partial T}{\partial H} \cdot \frac{H}{T}$$

⇒ CLCS:

$$S_G^T = \frac{1}{1+GH}$$

$$S_H^T = \frac{-GH}{1+GH}$$

⇒ OLCS: $S_G^T = 1$